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SCHOOL OF MATHEMATICS

COMPARISON OF DIFFERENT MODELS IN PRICING OF MICRO INSURANCE PRODUCTS FOR *BODA-BODA* OPERATORS

SAC 420: ***PROJECT IN ACTUARIAL SCIENCE***

DECLARATION

A RESEARCH PROJECT SUBMITTED IN FULFILLMENT FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE AT THE UNIVERSITY OF NAIROBI.

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DEDICATION

We dedicate this project to our supportive families for their substantial contributions.

And also to our supervisors for their guidance in the creation of this project.

May the Almighty reward you abundantly.

ABSTRACT

The micro-insurance industry is rapidly expanding in Kenya. New and differentiated products are being developed every day to meet the growing demands of the growing workforce as well as the evolving regulatory requirements. In this environment, insurers have to be innovative in developing, pricing and selling products to those employed in the informal sectors. The informal sector was previously ignored when it came to insurance. However, studies have shown that there is both a financial and social reward to providing such insurance services.

In this research, we focus on the '*boda-boda*' operators by pricing a product tailor-made to hedge against occupational risks faced by these operators. '*boda-boda*' operators are motorcycle riders who provide taxi services over short distances. We begin by reviewing the risks they face and checking if these risks are insurable. We then proceed to price the product using the traditional actuarial techniques- Empirical Bayes' Credibility Theory models I and II.

We then propose the use of the Black Scholes model to price the same product. Previous studies have shown that a short term insurance product such as a '*boda-boda*' product can be replicated as a European call option. We translated the parameters and assumptions of the original Black Scholes model to fit a *boda-boda* product and priced it.

Consequently, we compared the premiums under the three models and proposed a way forward for pricing products in the micro-insurance industry, particularly a product tailor-made for the '*boda-boda*' operators.

Table of Contents

INTRODUCTION

Background of the study

This project seeks to evaluate an insurance scheme tailor-made for low income motorcycle taxi drivers commonly referred to as *boda-boda* in Kenya.

Insurance penetration in Kenya currently stands at 3.1% of the gross domestic product (GDP). This low penetration is attributed to various factors including poor savings culture, low levels of disposable income and negative perception towards insurance (Insurance regulatory authority, 2014).

Micro insurance aims at enabling low income earners manage risks such as accident, illness, theft and fire in exchange for affordable premiums tailored to the needs, income, and nature of risks faced by buyers. Disability income insurance schemes are not readily available in Kenya as pure packages. Insurers that do underwrite disability risks normally package them as components of other insurance schemes, such as the Group Medical Scheme for outpatient and in-patient treatments. This ultimately drives up the premiums payable to access this service since the insured pays all other risks to access cover against loss of income due occupational risk.

Industry participants suggest that this is due to lack of demand for the product, and as such the scheme would not be a financially viable product for any company. Approximately 80% of the working populations are employed in the informal sector. Workers in the informal sector face a huge number of occupational hazards daily thus they provide a large target population for any company seeking to provide a micro-health insurance scheme.

In our report we focus on modeling claim amounts that may arise in a bid to offer an affordable policy to *boda-boda* operators against the occupational risks they face.

Statement of Problem

In an underperforming economy such as the Kenya economy, the majority of people are classified as low-income earners. This implies that they barely eke out a living. The problem is compounded by a seemingly low level of unemployment. The poor rate at which jobs are created does not even begin to match the rate at which competent labor joins the job market.

Therefore, some of the unemployed persons venture out into self-employment. It is under such a climate that motorcycle taxi drivers emerge. The need for an insurance scheme which caters for the needs of these low-income earners cannot be overemphasized.

Inevitably, motorcycle taxi drivers face risks connected to their daily activities. It may even be argued that their low-income status prods them to take numerous risks during their occupational activities. Their exposure to enhanced risk-taking invariably causes many of them to suffer losses. The relevance of the contribution made by low-income earners towards an economy is always an enduring issue. Consequently, it is of paramount importance that such low-income earners be kept in business.

Insurance plays a vital role in buttressing an economy by creating strong guarantees towards the continuity and general health of the economy. An economy without viable insurance schemes would crumble at the slight whim of market uncertainty. Granted, it is expected that an economy majorly composed of low-income earners would have to manifest a low level of insurance products and policies. This occurs due to the general negative perception that exists towards insurance. Since low-income earners focus on meeting their daily needs, they spare little thought for savings. Nevertheless, it is critical that an insurance scheme is set up which not only clears the barrier presented by imperceptions, but also overcomes the challenge created by a deplorable savings culture.

Objectives of our study

The general objective of the study is to price a product tailor-made to insure '*boda-boda*' operators against occupational risks they face, by comparing different valuation methods against each other.

Specific Objectives:

- To apply traditional actuarial techniques in valuing a '*boda-boda*' micro-insurance product.

- To translate the Black Scholes model to an actuarial model used to value the product under investigation.
- To propose a unified model to be used in the industry to price products in the micro insurance market.
- To check the extent to which each parameter affects the premium valuation.

Justification

Generally, low-income citizens such as *boda-boda* operators are often more exposed to such risks as death, illness, and higher occupational hazards as they are at a limited stage of development and have less access to formal insurance to protect them. Therefore, there is an increasing need to learn to insure the low wage citizens in developing countries and as such we focus on how to better the lives of '*boda-boda*' operators through offering suitable and affordable insurance against the occupational risks they face.

It estimated that motorcycles account for 70% of motor vehicles registered in Kenya each year (WHO[1] 2014). Road accidents are the third leading cause of death and this has a large cost to the country's gross national product. This proves there is a need for products such as this micro finance scheme to help cushion the drivers against their occupational risks and to increase their ability to cope with loss.

The greatest challenge for micro insurance schemes is finding the right balance between adequate protection and affordability to deliver real value to the insured. Data on needs, demand and risks is not easy to obtain, making product design difficult. Our project aims to develop a micro insurance scheme to helps '*boda-boda*' motorists manage unexpected events in return for payments proportional to the likelihood and level of a specific risk.

We seek to provide a customized product design that offers premiums that are small enough to be affordable, documentation that is reduced to a minimum and recommend delivery channels that reach out to this labor class.

[1] World Health Organization.

LITERATURE REVIEW

This section deals with previous works that have been done on, and related to our research topic. We briefly describe the research, the contributions and any limitations in a selection of previous works.

Use of pure premium models in automobile insurance market

Beomha Jee, an assistant professor of insurance at the Pennsylvania State University, compared the various pure premium models in the automobile risk classification system (Jee, 1989) . This research was based primarily on the fact that a classification system is necessary to group homogeneous risks and charges each group a premium commensurate to the average expected loss of its members. It used sample data (Massachusetts 1979-1980)[2] to compare predictive accuracy of alternative pure premium models. The study concluded that there is statistical evidence to support the empirical Bayes estimators based on credibility theory to predict future pure premiums. However, this study was limited in that there was limited availability of new data sets as well as it did not analyze in depth the possibility of heterogeneity within the risk classes[3].

Christian Biener evaluated the pricing problems in the micro insurance markets. (Biener, 2011) This study appreciated the strong growth rates in the micro insurance markets as well as the fundamental issues of pricing risk in these markets. It investigated conventional techniques as potential solutions for improving the pricing of insurance risk in micro insurance markets. The study concluded that credibility models can take advantage of available risks data, synthesize risk characteristics into a technical premium and update premiums when more loss experience becomes available over the contract period. In addition, the study recommended that bootstrap techniques can be used where only small samples of original loss data is available as prevalent in micro insurance. However, this study did not empirically test the methodologies suggested for the estimation of technical premiums.

[2] *Beomha Jee "The Journal of Risk and Insurance".*
[3] *Most studies encounter the same limitation*

Viability of a micro insurance product for *boda-boda* taxi drivers

Alma Cohen and Rajeev Dehejia investigated the incentive effects of automobile insurance, compulsory insurance laws and no-fault liability laws on driver behavior and traffic fatalities. (Cohen & Dehejia, 2004) This article aids the second part of our research project; the relevance of a micro insurance product in the motorcycle transport market. The evidence they presented indicated that the compulsory insurance rules in the USA deliver their intended effect that is reducing the incidence of uninsured motorists. This study is however limited in that it only focused on motorists in the USA over a relative short period 1970-1998. The dataset is limited because the taxi services evaluated are only the traditional car taxis available in the USA.

Designing new automobile insurance pricing systems, actuarial and social considerations

Daqing Huang and J. Tim Query assessed the social and actuarial considerations such as the living standards of the beneficiaries in question, the economic effects of the automobile industry as a whole to an economy and thus the importance of new insurance pricing systems. One of the objectives was to push the automobile insurance industry's business model adjustment forward. They focused on this includes balancing risks versus premiums, and supplies versus needs. The problems they encountered in designing the new pricing system were such as the organization of data, the treatment of large claims (two methods are utilized here; the claim value and percentage of claim frequency), the usage of loss distribution and the Bonus–Malus system which does not apply in our country.

They recommend rate smoothing be used where linear relationships between resultant risk premiums within a risk category fit a linear relationship. The actuary thus can smooth the rates to fit this linear relationship, making rates easier to quote. Other fitting models such as the generalized linear modeling (GLM) and the MAX model are reviewed as well as forecasting the target market's price which is the core competency of the for-profit venture. Loss development analysis estimates the number and value of claims not yet reported as well as adverse (or favorable) development on known claims, with each different type of risk, such as third party versus property claims requiring a separate development pattern.

The Black-Scholes Model

In their 1973 paper "The pricing of options and corporate liabilities" Fischer Black and Myron Scholes put up a theorem for valuing option contracts under ideal conditions in the market for the stock and for the option. This theorem came to be known as The Black Scholes Model, and has since been used in valuing option contracts.

Over time, the model has received criticism from different quotas. Key to the Black-Scholes model is the assumption that the underlying stock price moves randomly following a geometric Brownian motion. However, the stock price distribution does not conform strictly to this normality assumption. In addition, the assumption of a constant and known short term interest rate is also adopted for convenience and not strictly true. The focus on European contracts was designed to allow us ignore the potential influence of early exercise (Fortune and Peter, 1996).

Merton (1973) however shows that if there are no additional payments made during the lifetime of the option then it would be irrational for an investor to exercise an American option before the maturity date. The Black-Scholes model can therefore be used to evaluate American options based on non-dividend paying common stocks. He further modified the equations to account for both American and European style options as well as stochastic interest rates.

In "On Jumps in Common Stock Prices and Their Impact on Call Option Pricing," Ball, et al., performed tests on the Merton's Jump- diffusion model and the Black - Scholes model. Interestingly, they concluded that there were no operationally significant differences in the models.

Merton (1976) and Cox and Ross (1976) further modified the model to allow for discontinuous stock price movements.

Merton (1998) remarked that the influence of the Black-Scholes option theory in finance isn't limited to financial options traded in markets or even to derivatives in general. It can also be used to price and evaluate risk in a wide array of applications, both financial and non-financial.

Hong Boon Kyun (2004) applied the Black-Scholes warrant pricing model to the stock exchange of Malaysia. He concluded that despite the existence of strike price, time to maturity and variance biases in the model there were no significant differences between the market value of warrants and the Black-Scholes value of warrants.

METHODOLOGY

Empirical Bayes Credibility Theory

Credibility theory

Credibility Theory is a technique in actuarial science that is used in the estimation of next year's premium or claim frequency. Given the following information:

- × =An estimate of the expected aggregate claim or number of claims for the coming year based solely on data from the risk itself. X
- μ- An estimate of the expected aggregate claim or number of claims for the coming year based on collateral data, that is, data from risks similar (but not necessarily identical) to the risk being considered.

The credibility estimate for the aggregate claims or number of claims can be computed as:

$$P = Z\,X + (1 - Z)\mu$$

Where, *Z* is a figure between 0 and 1 referred to as the **credibility factor**.

Clearly, it is a measure of how much trust can be placed on data from the risk itself as an estimate next year's expected aggregate claims or number of claims.

This formula has come to be known as the credibility premium formula.

Bayesian credibility

At the core of the actuarial profession is the estimation of the credibility factor for different risks with time. For decades, Bayesian approaches have been used to estimate *Z*. This involves assuming a particular distribution for the claims and estimating a parameter θ.

The Bayesian estimate of θ as per a given loss function is then computed and the

credibility factor identified. The loss function provides a measure of how serious getting

the wrong estimate of *Θ* is. We can illustrate this using the Poisson/Gamma model.

The Poisson/Gamma Model

Suppose we need to estimate the claim frequency for a risk, *i.e.* the expected number

of claims in the coming year. This problem can be summarized as follows:

The number of claims each year is assumed to have a Poisson distribution with

Parameterλ.

Even though the value of λ is not known, it may be taken that λ has a

Gamma (α, β) distribution, from past experience. (The gamma distribution is a natural

Prior distribution to assume when claims follow a Poisson distribution).

Data from this risk are provided showing the number of claims arising in each of the past, say, *n* years.

This problem fits exactly into the Bayesian Statistics framework and can be formalized as:

Let the random variable *X* represent the number of claims in the coming year from a risk. The distribution of *X* depends on a fixed (but unknown) value of a parameter λ such that $X|\lambda$ has a Poisson (λ)distribution. The prior distribution of λ is given as Gamma (α, β).

We are also given claim frequencies for the past n years, $x_1, x_{2,\ldots,} x_n$ which, for convenience, will be denoted *x* .

The problem is now to find the estimate of λ given the observed values *x.* We can find the estimate with respect to quadratic loss, which is simply the mean of the posterior distribution of .

The PDF of λ is given as:

$$f(\lambda) = \frac{\beta \propto}{\Gamma\, \alpha} \lambda \wedge (\alpha - 1)\, e \wedge (-\beta\lambda) \qquad , \qquad \lambda > 0$$

The likelihood of incurring Xi claims in the *i-th* policy, *i* = 1, 2, …, n, is given by:

$$L = \prod \frac{e \wedge (-\lambda)\, \lambda \wedge (x)}{x!}\ \alpha\, e \wedge (-n\lambda)\, \lambda\ \wedge (\sum x)$$

Multiplying the likelihood function and the prior distribution yields the result that the posterior is a Gamma ($\sum X_i + \alpha$, n +β) distribution.

Therefore the Bayesian estimate of λ under quadratic loss is the mean of the posterior distribution that is:

$$\frac{\sum X + \alpha}{n + \beta}$$

This can be written in credibility form as

$$Z\ \frac{\sum X}{n} + (1 - z)\frac{\alpha}{\beta}$$

Where the credibility factor Z is given by: Z = $\frac{n}{n+\beta}$

The Normal/ Normal model

Let the aggregate claims X be independent and the unknown parameter θ is the true risk premium.

Assume $X_i | \theta$ be from the normal distribution with mean θ and variance δ^2 where the variance is known.

The prior distribution of θ is normal with mean μ and variance δ_0^2 where the mean and variance are known.

The posterior distribution is derived by the Bayesian method and is found to be normal with the following parameters:

$$Mean = \frac{\mu\sigma 2 + \ \sigma 2n\sum x/n}{\sigma 2 + n\sigma 2}$$

$$Variance = \frac{\sigma 2\sigma 2}{\sigma 2 + n\sigma 2}$$

The credibility risk premium is therefore given by:

$$E[\theta|X] = \frac{\sigma 2}{\sigma 2 + n\sigma 2}\,\mu + \frac{n\sigma 2}{\sigma 2 + n\sigma 2}\ {\textstyle\sum x}/_{n}$$

EMPIRICAL BAYES CREDIBILITY THEORY

This theory is a significant improvement to the traditional Bayesian credibility since no assumptions are made about the underlying distribution of the claim frequencies or sizes. Instead, it assumes that the variation within the claims from a given risk of interest and that from similar risk can be used to predict next year's change in the premium or claim frequency.

There are similarities between the EBCT models and Bayesian models. They include:

1. Risk parameter: Both approaches make use of an auxiliary risk parameter θ. In the Bayes models, we seek to determine θ and in the EBCT models, we seek to estimate m(θ).
2. Conditional claim distribution: In both the Bayes and EBCT models, the variables of interest $X_j | \theta$ are conditionally independent and identically distributed, given the value of θ.
3. Credibility formula: Using both approaches, the final formula for estimating the risk premium or claim frequency for a certain risk can be expressed.

EMPIRICAL BAYES CREDIBILITY THEORY: MODEL 1

We seek to estimate the premium or claim frequency for the subsequent year given historical data for the past n years.

Assumptions

1. The distribution of each X_j depends on some parameter θ , whose value is fixed but unknown.
2. The X_j's are independent and identically distributed random variables.

Assume $\underline{X}$ are the aggregate claims in successive years and the parameter of interest is θ.

Let the mean $m(\theta) = E(X_j | \theta)$ be the risk premium and $S^2(\theta) = Var(X_j | \theta)$

We need to determine the Bayes estimate E [m (θ) |X] = a + b x

Further, we need to estimate E [m (θ)], Var [m (θ)] and E [S^2 (θ)].

For convenience, we shall put the data in the following form:

	1	**2**	**…**	**n**	**PARAMETER**	**Mean**	**Sample variance**
1	X_{11}	X_{12}		X_{1n}	Θ_1		
2	X_{21}	X_{22}		X_{2n}	Θ_2		
..							
N	X_{N1}	X_{N2}		X_{Nn}	θ_N		

The estimate of m(θ) is (1-Z) E[m(θ)] + Z X where X is the sample mean and $Z = \frac{n}{n+\frac{E[\,S2\,(\theta)]}{Var\,[\,m(\theta)]}}$

From the above table:

Let X_{ij} be the aggregate claim or claim frequency for risk number i in year j. N is the number of risks in the portfolio and n is the number of years over which data is available.

We define the statistics:

$X_i = \frac{1}{n} \sum Xij$ (Row mean)

$X = \frac{1}{N} \sum Xi = \frac{1}{N}\frac{1}{n} \sum\sum Xij$ (Grand mean)

The credibility premium for each risk is given by:

$$Z\,X\,i + (1 - Z)\,E[m(\theta)]$$

More precisely, $m(\theta_i)$ = Row mean and $s^2(\theta) = \frac{1}{n-1} \sum(Xij - Xi) \wedge 2$

The best linear parameter estimates are:

E[m(θ)] = Grand mean

$E[s^2(\theta)] = \frac{1}{N} \sum \left\{\frac{1}{n-1} \sum(Xij - Xi)2\right\}$

$Var\,[m(\theta)] = \frac{1}{N-1} \sum(Xi - X)2 - \frac{1}{Nn} \sum\{\frac{1}{n-1} \sum(Xij - Xi)2\}$

Using the above estimates, we can compute the credibility factor and therefore obtain the risk premium for the next year.

Empirical Bayes Credibility Theory Model II

EBCT model I is barely used in practice due to the major drawback that it does not take into account the volume of business written in each of the past n years. For most insurance companies in Kenya, the number of policyholders has been increasing steadily with time. To take this into account, we turn to EBCT model II, which weights the aggregate annual claims by the volume of risk in each year.

Specification

The problem once again is to estimate the pure premium or claim frequency in the following year.

Let $Y_1, Y_2, \ldots, Y_n$ be random variables representing the aggregate claims (or number of claims) in n successive years for a risk. Our task will be to estimate Y_{n+1}.

We now introduce a new parameter known as the risk volume, P_j, which gives the amount of business written in year j. The value of P_{n+1} is assumed to be known.

Define a new sequence of random variables $X_1, X_2, \ldots, X_{n+1}$ such that:

$$X_j = \frac{Y_j}{P_j}, \qquad j = 1, 2, \ldots$$

By definition, X_j is the random variable representing the aggregate claims (or number of claims) in year j standardized to remove the effect of different levels of business in different years.

Assumptions

1. The distribution of each X_j depends on the value of a parameter θ, whose value is unknown but the same for each j.

2. Given θ, the X_j*'s* are independent, but not necessarily identically distributed.

3. $E(X_j|\,\theta)$ does not depend on j.

4. $P_j\, var(X_j|\,\theta)$ does not depend on j.

We then define:

$$m(\theta) = E\left(X_j|\theta\right)$$

$$S^2(\theta) = P_j var(X_j|\theta)$$

Such that:

$$E\left(Y_j\right) = P_j m(\theta)$$

$$var\left(Y_j\right) = P_j S^2(\theta)$$

$$E\left(X_j\right) = m(\theta)$$

$$P_j var\left(X_j\right) = S^2\,(\theta)$$

The Credibility Premium

We now need to estimate the mean (given θ) of Y_{n+1}, *i.e.* $P_{n+1}\, m(\theta)$.

The best linear estimator of $m(\theta)$ given $\underline{X}$ is:

$$m(\hat{\theta}) = \frac{\dfrac{E[m(\theta)]E[S^2(\theta)]}{var[m(\theta)] + \sum_{j=1}^{n} Y_j}}{\sum_{j=1}^{n} P_j + \dfrac{E[S^2(\theta)]}{var[m(\theta)]}}$$

The results of EBCT model II can be summarized as:

$$m(\theta) = Z\bar{X} + (1 - Z)E[m(\theta)]$$

where:

$$\bar{X} = \frac{\sum_{j=1}^{n} P_j X_j}{\sum_{j=1}^{n} P_j} = \frac{\sum_{j=1}^{n} Y_j}{\sum_{j=1}^{n} P_j}$$

and

$$Z_i = \frac{\sum_{j=1}^{n} P_{ij}}{\sum_{j=1}^{n} P_{ij} + \dfrac{E[S^2(\theta)]}{var[m(\theta)]}}$$

Once we obtain the estimate of X_{n+1}, we simply multiply the result by P_{n+1} to obtain the estimate of Y_{n+1}.

Parameter Estimation

Once again the bulk of the work in the EBCT model II lies with estimating the parameters of the model.

Let Y_{ij} be the aggregate claim or number of claims for risk i, $i=1,2,\ldots,N$, in year j, $j=1,2,\ldots,n$. Further let P_{ij} be the amount of business written for risk i, $i=1,2,\ldots,N$, in year j, $j=1,2,\ldots,n$.

We define more notations as:

$$\bar{P_i} = \sum_{j=1}^{n} P_{ij}$$

$$\bar{P} = \sum_{i=1}^{N} \bar{P_i}$$

$$P^* = \frac{1}{Nn - 1} \sum_{i=1}^{N} \bar{P_i}\left(1 - \frac{\bar{P_i}}{\bar{P}}\right)$$

$$\bar{X_i} = \sum_{j=1}^{n} \frac{P_{ij} X_{ij}}{\bar{P_i}}$$

$$\bar{X} = \sum_{i=1}^{N} \frac{\bar{P}_i \bar{X}_i}{\bar{P}} = \sum_{i=1}^{N} \sum_{j=1}^{n} \frac{P_{ij} X_{ij}}{\bar{P}}$$

The unbiased estimators are given as:

$$E[m(\theta)] = \bar{X}$$

$$E[S^2(\theta)] = \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{1}{n-1} \sum_{j=1}^{n} P_{ij} \left(X_{ij} - \bar{X}_i \right)^2 \right\}$$

$$var[m(\theta)] = \frac{1}{P^*} \left\{ \frac{1}{Nn-1} \sum_{i=1}^{N} \sum_{j=1}^{n} P_{ij} \left(X_{ij} - \bar{X}_i \right)^2 \right\} - E[S^2(\theta)]$$

Specification

The problem once again is to estimate the pure premium or claim frequency in the following year.

Let $Y_1, Y_2, \dots, Y_n$ be random variables representing the aggregate claims (or number of claims) in n successive years for a risk. Our task will be to estimate Y_{n+1}.

We now introduce a new parameter known as the risk volume, P_j, which gives the amount of business written in year *j*. The value of P_{n+1} is assumed to be known.

Define a new sequence of random variables $X_1, X_2, \dots, X_{n+1}$ such that:

$$X_j = \frac{Y_j}{P_j}, \qquad j = 1, 2, \dots$$

By definition, X_j is the random variable representing the aggregate claims (or number of claims) in year *j* standardized to remove the effect of different levels of business in different years.

Assumptions

1. The distribution of each X_j depends on the value of a parameter θ, whose value is unknown but the same for each *j*.

2. Given θ, the X_j's are independent, but not necessarily identically distributed.

3. $E(X_j | \theta)$ does not depend on *j*.

4. $P_j \, var(X_j | \theta)$ does not depend on *j*.

We then define:

$$m(\theta) = E\left(X_j | \theta\right)$$

$$S^2(\theta) = P_j var(X_j | \theta)$$

Such that:

$$E\left(Y_j\right) = P_j m(\theta)$$

$$var\left(Y_j\right) = P_j S^2(\theta)$$

$$E(X_j) = m(\theta)$$

$$P_j var(X_j) = S^2\,(\theta)$$

The Credibility Premium

We now need to estimate the mean (given θ) of Y_{n+1}, *i.e.* $P_{n+1}\, m(\theta)$.

The best linear estimator of $m(\theta)$ given $\underline{X}$ is:

$$m(\hat{\theta}) = \frac{\dfrac{E[m(\theta)]E[S^2(\theta)]}{var[m(\theta)] + \sum_{j=1}^{n} Y_j}}{\sum_{j=1}^{n} P_j + \dfrac{E[S^2(\theta)]}{var[m(\theta)]}}$$

The results of EBCT model II can be summarized as:

$$m(\theta) = Z\bar{X} + (1 - Z)E[m(\theta)]$$

where:

$$\bar{X} = \frac{\sum_{j=1}^{n} P_j X_j}{\sum_{j=1}^{n} P_j} = \frac{\sum_{j=1}^{n} Y_j}{\sum_{j=1}^{n} P_j}$$

and

$$Z_i = \frac{\sum_{j=1}^{n} P_{ij}}{\sum_{j=1}^{n} P_{ij} + \dfrac{E[S^2(\theta)]}{var[m(\theta)]}}$$

Once we obtain the estimate of X_{n+1}, we simply multiply the result by P_{n+1} to obtain the estimate of Y_{n+1}.

Parameter Estimation

Once again the bulk of the work in the EBCT model II lies with estimating the parameters of the model.

Let Y_{ij} be the aggregate claim or number of claims for risk *i, i=1,2,…,N*, in year *j, j=1,2,…,n*. Further let P_{ij} be the amount of business written for risk *i, i=1,2,…,N*, in year *j, j=1,2,…,n.*

We define more notations as:

$$\bar{P_i} = \sum_{j=1}^{n} P_{ij}$$

$$\bar{P} = \sum_{i=1}^{N} \bar{P_i}$$

$$P^* = \frac{1}{Nn - 1}\sum_{i=1}^{N} \bar{P_i}\left(1 - \frac{\bar{P_i}}{\bar{P}}\right)$$

$$\bar{X}_i = \sum_{j=1}^{n} \frac{P_{ij} X_{ij}}{\bar{P}_i}$$

$$\bar{X} = \sum_{i=1}^{N} \frac{\bar{P}_i \bar{X}_i}{\bar{P}} = \sum_{i=1}^{N} \sum_{j=1}^{n} \frac{P_{ij} X_{ij}}{\bar{P}}$$

The unbiased estimators are given as:

$$E[m(\theta)] = \bar{X}$$

$$E[S^2(\theta)] = \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{1}{n-1} \sum_{j=1}^{n} P_{ij} \left(X_{ij} - \bar{X}_i \right)^2 \right\}$$

$$var[m(\theta)] = \frac{1}{P^*} \left\{ \frac{1}{Nn-1} \sum_{i=1}^{N} \sum_{j=1}^{n} P_{ij} \left(X_{ij} - \bar{X}_i \right)^2 \right\} - E[S^2(\theta)]$$

These parameters will then be inserted in formula 3.23 to obtain the credibility factor, which will then be used in formula 3.21 to estimate the risk premium for the next year.

The Black-Scholes Model

Fischer Black and Myron Scholes formulated a theorem for valuing option contracts under ideal conditions in the market for the stock and for the option in 1973. This theorem came to be known as The Black Scholes Model, and has since been used in valuing option contracts.

However, the model has received criticism from different quotas. The major assumption of the Black-Scholes model involves the geometric Brownian motion of the underlying stock price. However, the stock price distribution does not conform strictly to this normality assumption. In addition, the assumption of a constant and known short term interest rate is also adopted for convenience. Nevertheless, the interest rate is not usually constant, even over a short period of time. It can also be used to price and evaluate risk in a wide array of applications, both financial and non-financial. Hong Boon Kyun (2004) applied the Black-Scholes warrant pricing model to the Malaysian stock exchange. He concluded that despite the existence of strike price, time to maturity and variance biases in the model, there were no significant differences between the market value of warrants and the Black-Scholes value of warrants.

The Model

Let f (t, S_t), be the price of a call option at time *t*, given:

i.) The current share price, S_t
ii.) The time to maturity, T>t
iii.) The exercise price, K
iv.) The risk free rate, r
v.) The volatility of the share price, σ.

The model proposes that:

$$f(t, S_t) = S_t \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2)$$

Where

$$d_1 = \frac{\log\left(\frac{S_t}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = d_1 - \sigma\sqrt{T-t},$$

Assumptions of the Black-Scholes Model

There are a few assumptions that need to be considered when calculating the price of a call option over a period with the Black Scholes model:

1. The short-term interest rate is known and is constant through time.
2. The option is 'European' in that it can only be exercised at maturity
3. The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. As a result, the distribution of the possible stock prices at the end of any infinite time interval is log normal and the variance rate of return on the stock is constant.
4. The stock pays no dividends or other distributions.
5. There are no transaction costs in buying or selling the stock or the option.
6. It is possible to borrow any fraction of the price of a security to buy it or to hold it at the short term interest rate.
7. There are no penalties to short selling.

Limitations of the Black-Scholes Model

The Black-Scholes model has been widely used in the pricing of option contracts. However, like any other financial model, it is not immune to shortcomings. In this section, we analyze these drawbacks by challenging the major assumptions underlying the model as follows:

a) **Volatility is constant over time**: Volatility is a measure of how much the value of a given stock is expected to move within the near future. While it is likely to be relatively constant in the short term, it can never be constant in the long run. In practice, large changes in price tend to be followed by even larger changes and vice-versa, resulting in a property known as volatility clustering. This is perhaps why more advanced option valuation models swap the Black-Scholes' constant volatility with estimates generated from stochastic processes.

b) **People can't consistently predict the direction of the market or of an individual stock**: The model assumes that stock prices follow a symmetric random walk, in that at any given time, the price can move up or down with equal probability. Unfortunately, stock prices are subject to several economic factors that may not be assigned the same probability. In addition, the martingale property of a Brownian motion explains that the stock price at time t+1 is independent from that at time t.

c) **The returns of log-normally distributed stock prices follow a normal distribution**: While this remains a reasonable assumption in the real world, it struggles to fit observed financial data. For instance, asset returns have infinite variance as well as heavy tails.

d) **Interest rates are known and constant**: The Black-Scholes model is built around a risk-free rate of interest, which remains constant during the term of the contract. However, these rates can change significantly in periods of heightened volatility.
e) **The underlying stock doesn't pay dividends during the life of the option**: The original Black-Scholes model does not make allowance for dividends, despite the fact that most companies pay out dividends. One way to work around this problem is to subtract the discounted value of future dividend from the stock price.
f) **No commissions or transaction costs**: The model assumes that no fees is charged for the buying and selling of options and that there are no barriers to trading. However, in practice, stock brokers charge varying rates.
g) **Option can only be exercised at expiration date**: American-style options, which can be exercised on any day before or at maturity, may not be valued accurately by the model.
h) **Markets are perfectly liquid and any amount of options can be purchased at any** time: This is hardly practical as investors are always limited by the much they can invest, company policies and wishes of the sellers to sell. In addition, it may not be possible to purchase or sell fractions of options.

In comparing the different models, this paper may address some of these fall backs the model has on pricing the *boda-boda* micro insurance product we are proposing.

Adaptation of the Black-Scholes Model to *boda-boda* Insurance

Correlation between boda-boda insurance schemes and option contracts

A major part of our study revolves around how the Black-Scholes model that has been widely used in pricing derivatives can be applied in the *boda-boda* insurance market. Before we embark into the translation of model parameters, it is critical to consider the similarities between options and *boda-boda* insurance.

1. *Compensation*

In the event that the unexpected situation arises, compensation must be paid in either case. When unexpected price changes occur, the buyer of an option will execute it, receiving a compensation equivalent to the difference between the market price and the strike price. In case of a risk whose treatment cover exceed the set minimum, the buyer of *boda-boda* insurance cover will be compensated. In case the covered event does not materialize, the buyer forfeits the premium in both cases.

1. *Payment of Premium*

In both cases, the purchaser of the contract has to part with a premium. The buyer of an option pays a premium at the time of purchase to obtain the desired hedge while policyholders pay premiums at the start of the period they intend to be covered.

2. *Hedging Operations*

Firstly, *boda-boda* insurance schemes and option contracts are both hedging operations. While options cover agents against unexpected price movements, insurance covers policyholders against unexpected illnesses or accidents.

3. Hedge Period

The timing of both insurance and option contracts is usually short. In the current financial market, options are rarely contracted for periods longer than one year. Similarly, most *boda-boda* insurance products offer one year coverage.

Translation of Parameters

Suppose a hypothetical individual intends to purchase *boda-boda* insurance cover. We also assume the individual's risk will generate 3 payments in the course of the year, at times 1, 2, 3, and that the accumulated cost will exceed the deductible. Without insurance, the individual will have to foot the entire annual treatment cost if they suffered an occupational risk. However, with insurance, there will be a maximum cost for the individual.

In nominal terms, the total cost paid out by this individual will be the deductible plus the premium amount paid at the start of the year. In this case, the cost minus insurance will be more than when the individual is insured. So how does this resemble the purchase of an option?

When purchasing a European call option, the buyer guarantees a maximum price for buying the underlying asset when the contract matures. In case the market price is lower than the strike price, the buyer is under no obligation to exercise the call option. If he/she wishes to purchase the asset, the market price has to be paid. The total cost of the asset will therefore be sum of the market price and the premium paid at the time of purchasing the option.

On the contrary, if the market price exceeds the strike price, the buyer of the option will exercise the right and only pay the strike price. The total cost paid will now be the strike price and the initial premium. In both hedge schemes, the buyer is guaranteed a maximum cost to be incurred when an unexpected event occurs, and has to pay a premium for that benefit.

The essential elements of an option contract are:

1. Buyer
2. Writer
3. Expiration date
4. Strike price
5. Premium
6. Underlying asset

In the case of a *boda-boda* insurance scheme, the buyer is the prospective policyholder while the writer of the contract is the insurance company providing the cover. The strike price here will be the amount of the deductible. This can be defined as the lower limit beyond which the insurance company will foot the treatment cost.

The most important part of an option, and a derivative by extension, is the underlying asset. In this valuation model, we define the underlying asset as the accumulated treatment cost in the year.

The expiration date will be defined as the end of the year and the premium for the "option" is the premium paid to the insurance company at the policy's inception. At expiration date, the contract will be settled by differences.

In this coverage pattern, in case the accumulated cost of treatment at the end of the year(expiration date) are lower than the amount of the deductible, the individual will not have exercised their right and will therefore assume all the payments. The total cost for the patient will be the sum of the annual treatment cost and the premium paid at the start of the year.

In the event that the accumulated treatment costs exceed the deductible, the insured individual will exercise the option. The individual now has the right to "buy" the accumulated treatment costs at the price equal to the deductible amount. Since the "market price" (the accumulated treatment cost) is higher than the amount of the deductible, the individual will exercise their right and "buy" the accumulated cost of treatment at the strike price, which is the deductible. The insurer will simply pay the difference between the accumulated treatment cost (the market price) and the deductible (the strike price). The contract is therefore settled by differences.

This means that the insured individual will only assume treatment costs lower than the deductible. The total cost on their part will be the sum of the premium paid at the outset and the deductible.

Clearly, an option contracts and the insurance contract can achieve the same coverage pattern. The following table summarizes the variables of *boda-boda* insurance policies that directly translate to our model:

	OPTIONS CONTRACT	*BODA-BODA* INSURANCE SCHEME
Buyer of the contract	Buyer of an option	The insured
Writer of the contract	Writer of an option	Insurance company
Premium	Premium payable upfront	Premium payable by insured
Underlying asset	Security's price	Accumulated treatment cost
Strike Price	Exercise price	Deductible
Time	Time to maturity	End of year

Table 3.1: Translation of the Black-Scholes parameters to a boda-boda insurance scheme

Therefore the price at the outset of a one-year *boda-boda* insurance cover, P(S), given:

i.) The accumulated cost of treatment is S,
ii.) The time to maturity is 1 year,
iii.) The amount of the deductible is K,
iv.) The risk free rate is r,
v.) the volatility of the cost of treatment is σ,

is;

$$P(S) = S\Phi(d_1) - Ke^{-r}\Phi(d_2)$$

where,

$$d_1 = \frac{\log\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)}{\sigma},$$

$$d_2 = d_1 - \sigma$$

Parameter Estimation

Volatility

Volatility is the measure by which a financial variable fluctuates during a period relative to a central trend or drift. Option pricing models require an estimate of expected volatility as an assumption because an option's value is dependent on potential underlying assets return over the period. The higher the volatility the higher the returns on the underlying asset are expected to vary either up or down.

From the Black-Scholes model:

$$S_T = S_t e^{\left(r - \frac{\sigma^2}{2}\right)(T-t) + \sigma(W_T - W_t)}$$

$$\frac{S_T}{S_t} = e^{\left(r - \frac{\sigma^2}{2}\right)(T-t) + \sigma(W_T - W_t)}$$

$$\ln\left(\frac{S_T}{S_t}\right) = \left(r - \frac{\sigma^2}{2}\right)(T - t) + \sigma\sqrt{T - t}^{\,*} z_1$$

Where; $\left(r - \frac{\sigma^2}{2}\right)(T - t)$ is trend and $\sigma\sqrt{T - t}^{\,*} z_1$ is white noise.

Letting T-t be one day gives

$$\ln\left(\frac{S_{t+1}}{S_t}\right) = \left(r - \frac{\sigma^2}{2}\right) + \sigma^* z_1$$

Define $R_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$. Therefore:

$$\bar{R} = \frac{1}{n}\sum_{i=1}^{n} R_i = \left(r - \frac{\sigma^2}{2}\right)$$

$$Var(R) = \frac{1}{n-1}\sum_{i=1}^{n}(R_i - \bar{R})^2 = \sigma^2$$

Therefore the volatility, σ, is the standard deviation of R

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(R_i - \bar{R})^2 = \sigma^2}$$

This standard deviation must be annualized to make it suitable for use in valuation

Let σ* be the annualized volatility

$$\sigma^* = \sigma\sqrt{Sample\ frequency}$$

Deductible (Strike Price)

The deductible will be set as the lowest amount that an insurer will be willing to cover. This will always vary across different insurance policies.

Time

The time component of our model will be taken as the period over which the policyholder will be covered. *Boda-boda* insurance schemes in Kenya are usually 1 year contracts so far.

Assumptions of the model

For the above adaptation of the Black-Scholes model to be valid in pricing *boda-boda* insurance benefits, the following assumptions are made:

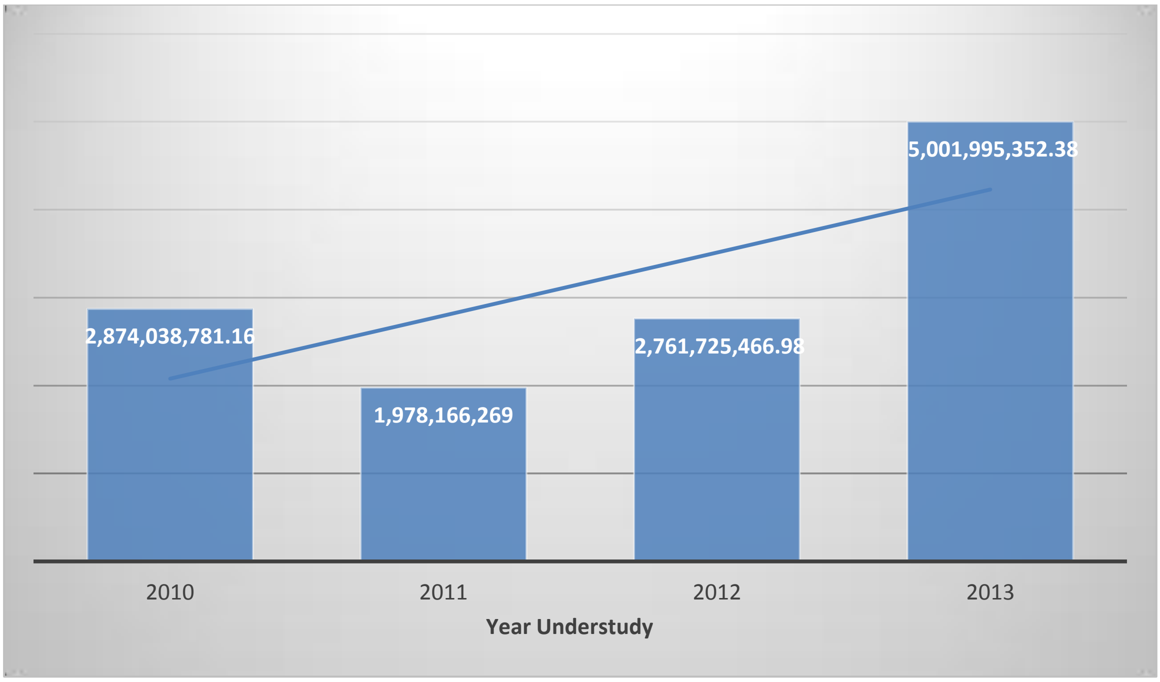
1. There are no costs associated with purchasing insurance cover and no taxes. In addition, the purchaser is free to choose the amount of the deductible.
2. There are no more additional costs apart from the accumulated treatment costs, which is our underlying asset.
3. All policies with the same extent of benefit have the same premium amount i.e. for the same premium; an individual can't obtain more cover from the same market.
4. There exists a risk-free rate of interest that is assumed to hold constant during the term of cover.
5. Policies can be purchased at any time in the course of the year.
6. There are no sudden increments in the amount of the deductible.
7. Individuals prefer more cover to less cover and agree on the variance of the deductible.

Data Analysis

Boda-boda insurance Trends in Kenya

Empirical Bayes Credibility Theory was used to determine the credibility premium for the year 2014. Due to the unavailability of information in the year 2014 a generalized linear model was used to extrapolate the EBCT of that year.

The claim amounts were separated into years and the individual claims in each year were summarized as follows:



Graph 1.1: ***Claim Amounts Trends for the Period 2011-2013***[4]

The claim amount is at an all-time high in the year 2013 due to the increase in capacity of the micro motor insurance industry and also this can be attributed to an increase in accidents rates of the *boda-boda* operators thus the need to increase of micro motor insurance capacity is in imperative. According to Table 1.1, the claim amounts increase with time hence the claim amounts might also increase with time.

Credibility Premium

EBCT1

The following was generated from the data;

Table 1.1: Summary of Mean & Variance of the Individual Years for the Period 2010-2013

YEAR	$\bar{y}_i$	s_i^2	$(\bar{y}_i-\bar{y})^2$
2010	31,922.10	2,874,038,781.16	25,982,550,080.21
2011	51,225.21	1,978,166,268.61	20,132,183,410.48

[4] Source, Author(2015)

2012	53,974.81	2,761,725,466.98	19,359,474,609.44
2013	55,991.01	5,001,995,352.38	18,802,478,659.77
TOTAL	193,113.14	12,615,925,869.12	84,276,686,759.90

Table 1.1: ***Summary of Mean & Variance of the Individual Years for the Period 2010-2013***

- ✓ $E\{M(\Theta)\} = \frac{\sum_i \bar{y}_i}{4}$

$$= \frac{193{,}113.14}{4}$$

$$= \underline{\underline{48{,}278.29}}$$

This is the overall mean, which was obtained by finding the average of the means for each fleet.

- ✓ $E\{S^2(\Theta)\} = \frac{\sum_i (s_i^2)}{4}$

$$= \frac{12{,}615{,}925{,}869.12}{4}$$

$$\underline{\underline{= 3{,}153{,}981{,}467.28}}$$

This is the average of the variance of each claim amounts in a given year for the period 2010-2013.

- ✓ $VAR\{M(\Theta)\} = \frac{\sum_i (\bar{y}_i - \bar{y})^2}{3} + \frac{E\{S^2(\Theta)\}}{596}$

$$= \frac{84{,}276{,}686{,}759.90}{3} + \frac{3{,}153{,}981{,}467.28}{596}$$

$$\underline{\underline{= 28{,}090{,}905{,}941.16}}$$

This is the variance between the individual mean claims in the given years understudy.

✓ $$Z=\left\{\frac{n}{n+\frac{E\{S^2(\Theta)\}}{VAR\{M(\Theta)\}}}\right\}$$

$$=\left\{\frac{596}{595+\frac{3,153,981,467.28}{28,090,905,941.16}}\right\}$$

Z =0.99981165

This is the credibility factor of the *boda-boda* micro motor insurance.

The credibility premium for the year, 2014= 67,018.92

2015= 74,514.15.

Therefore, given the past claims and previous risks the average premium is going to be Ksh. 67,018.92 in 2014 and Ksh. 74,514.15 in 2015.

Graph 1.2: ***Summary of the Credibility Premium for the Period 2010-2013***

EBCT II

Consider the table below;

Volume of business P(*ij*)	
Policy	**Year(j)**

type(*i*)	2010	2011	2012	2013	2014	current year
Policy A	85	254	316	303	318	564
Policy B	212	485	691	597	697	1077

Table 1.2: ***Summary Volume of Business for Period 2010-2014***

if the claims per unit volume, $X_{ij} = \frac{Y_{ij}}{P_{ij}}$, we generate the table below;

	Total claims per unit volume X_{ij}				
	Year j				
Policy	**2010**	**2011**	**2012**	**2013**	**2014**
Policy A	31779.341	35381.992	49193.45	48628.88	50266.52201
Policy B	514533.3	261270.72	53556.99	55897.22	77574.88379

Table 1.3: ***Values of X_{ij} from table 1.4***

Hence,

- $E\{M(\Theta)\} = \underline{840948.8}$
- $E\{S^2(\Theta)\} = \frac{1.94*10^{13}+8.23*10^{14}}{10}$

  $= \underline{84{,}244{,}000{,}000{,}000}$
- $VAR\{M(\Theta)\} = \frac{\frac{7.98*10^{14}+1.38*10^{19}}{19} - 8.4*10^{13}}{49.389389}$

  $\underline{= 615{,}265{,}339{,}802.52}$
- $Z_1 = \frac{1276}{1276+\frac{84{,}244{,}000{,}000{,}000}{615{,}265{,}339{,}802.52}}$

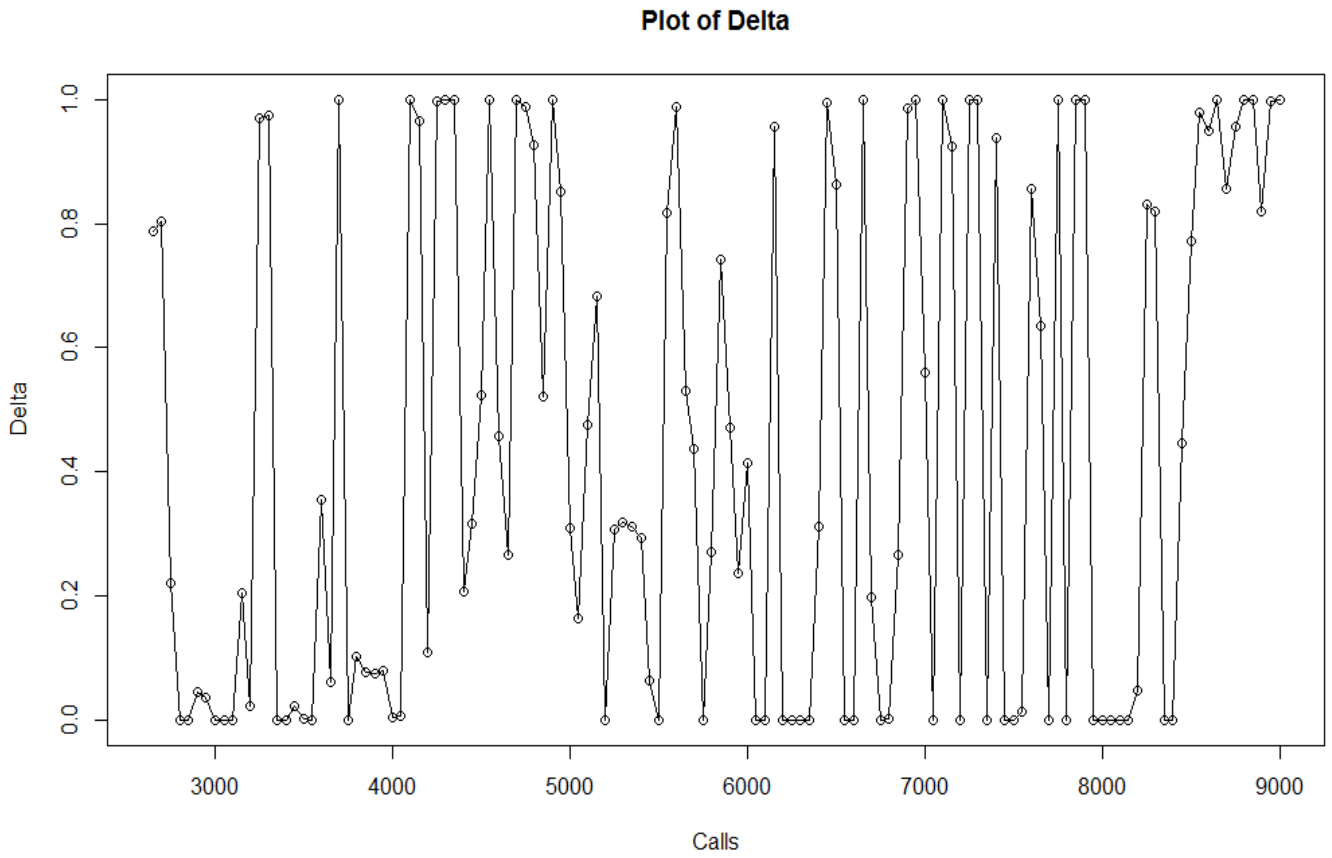
  $\underline{=0.903092359}$

Thus the risk premium is <u>53,935.</u>

Black-Scholes Model

The call option pricing basically is influenced by the delta(Δ) which depends on a normal distribution and it is a derivative from the Black-Scholes formula with respect to change in stock prices at any given time. The delta measures the sensitivity of the option price with respect to the stock price.

$\Delta = \partial C/\partial S$ which is how the delta is expressed in terms of partial differentiation. From the delta graph, it can be observed that delta is very sensitive to changes in volatility which is influenced by the call premiums as shown below.



From the general formula which is expressed (in R) as:

$C=SN(d_1)-N(d_2)Ke^{-rt}$

Where d_1=(ln(S/K) + (r + 0.5*sigma^2/2)t)/(sigma*squareroot(t))

And $d_2=d_1$-(sigma*squareroot(t))

C=call premium

S=current stock price

K=option striking price

N=cumulative standard normal distribution

And t=time to maturity, while r=risk free rate of interest and sigma=standard deviation.

Finally e=exponential term and ln=natural logarithm.

Consider t=365 days in a year of a policy, sigma=0.016935541, the two highest deltas from the graph $N(d_1)=0.8$ and $N(d_2)=1.0$. If we have S=150000 and K=70000 with the risk free rate, r=0.045.

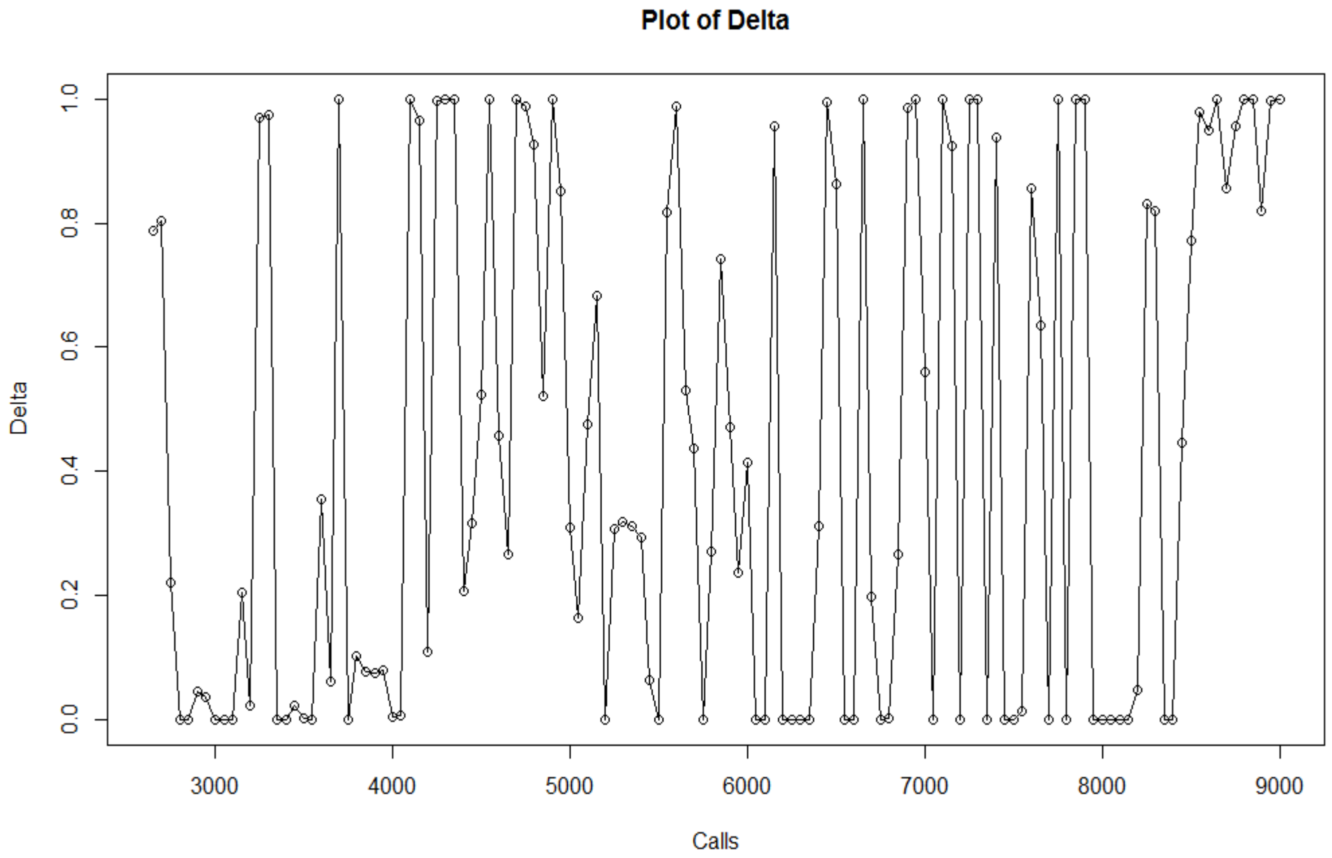
We get the call premium=54,928 and hence a conclusion that given the necessary parameters, we can predict the pricing of the premiums of the insurance policy.

Validity of Black-Scholes model

The black schools model is sensitive to three other particular variables which are Vega, Rho and Gamma. The Vega is a derivative function from the pricing function with respect to changing standard deviation. The Rho function is a derivative from the pricing function with respect to the varying interest rates while the Gamma function is the second derivative from the pricing function with respect to the changing strikes. When these functions are plotted against the varying prices (calls), they have an inverse relationship and therefore the model is Black-Scholes Model

The call option pricing basically is influenced by the delta which depends on a normal distribution and it is a derivative from the Black-Scholes formula with respect to change in stock prices at any given time. The delta measures the sensitivity of the option price with respect to the stock price.

$\Delta = \partial C/\partial S$ which is how the delta is expressed in terms of partial differentiation. From the delta graph, it can be observed that delta is very sensitive to changes in volatility which is influenced by the call premiums as shown below.



From the general formula which is expressed as:

$C=SN(d_1)-N(d_2)Ke^{-rt}$

Where $d_1=(\ln(S/K) + (r + 0.5*sigma^2/2)t)/(sigma*squareroot(t))$

And $d_2=d_1-(sigma*squareroot(t))$

C=call premium

S=current stock price

K=option striking price

N=cumulative standard normal distribution

And t=time to maturity while r=risk free rate of interest and sigma=standard deviation

Finally e=exponential term and ln=natural logarithm.

Consider t=365 days in a year of a policy, sigma=0.016935541, the two highest deltas from the graph $N(d_1)$=0.8 and $N(d_2)$=1.0 . If we have S=150000 and K=70000 with the risk free rate, r=0.0002.

We get the call premium=54,928 and hence a conclusion that given the necessary parameters, we can predict the pricing of the premiums of the insurance policy.

VALIDITY OF BLACK-SCHOLES MODEL

The Black Scholes model has the original formula as shown below:

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right]$$

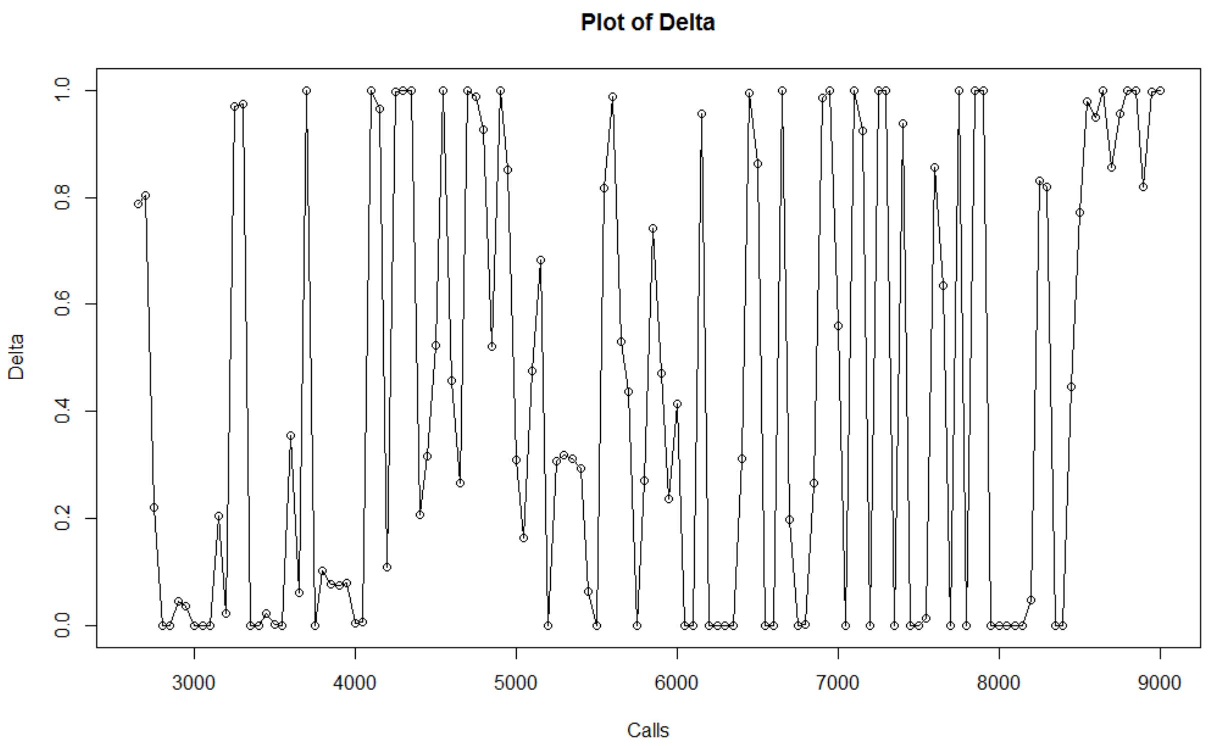
$$d_2 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)\right]$$

$$= d_1 - \sigma\sqrt{T-t}$$

Consider r=0.045, sigma=0.016935541, N(d1)=0.8 and N(d2)=1.0 as obtained from the delta graph, S=100000 and K=70000

Thus the Call price is, C=79999.995

Which can be approximated as 80000 as the premium price.



With respect to the original formula, the call price has a changing variable due to sensitivity giving rise to the Delta, Gamma, Vega, Theta and Rho. The new functions are as shown below.

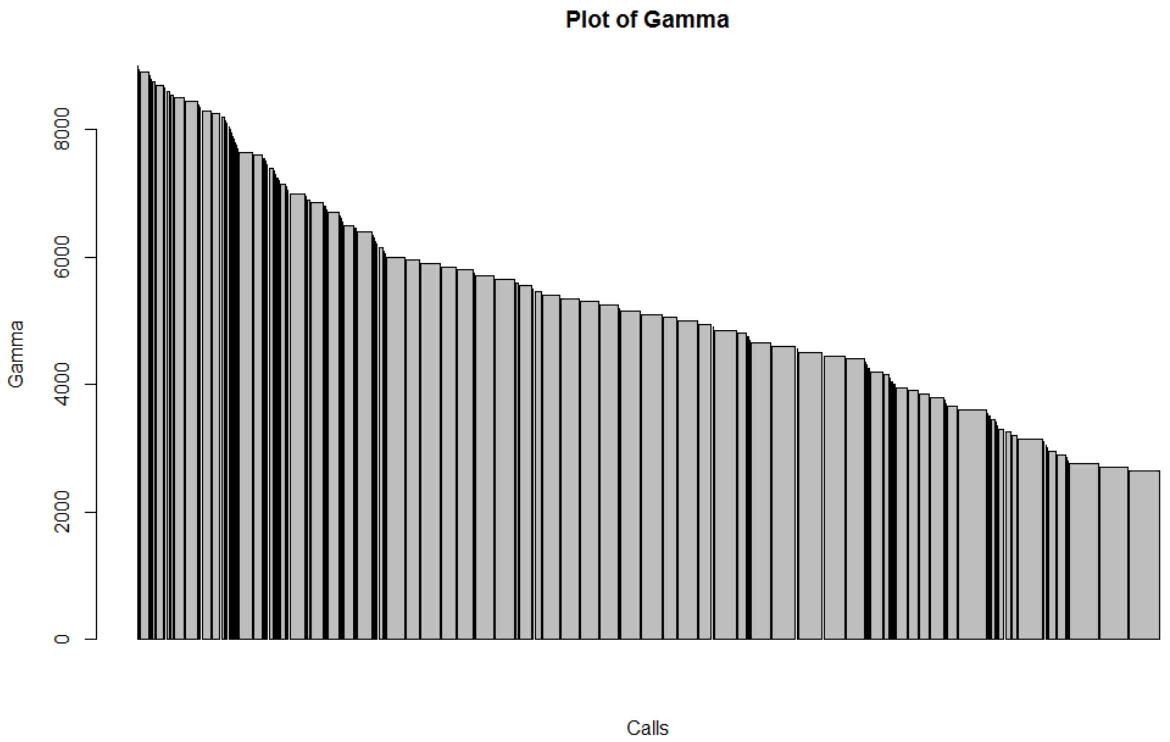
Delta

$$\frac{\partial C}{\partial S} = N(d_1)$$

As shown from the above graph, the selected N(d1)=0.8

Gamma

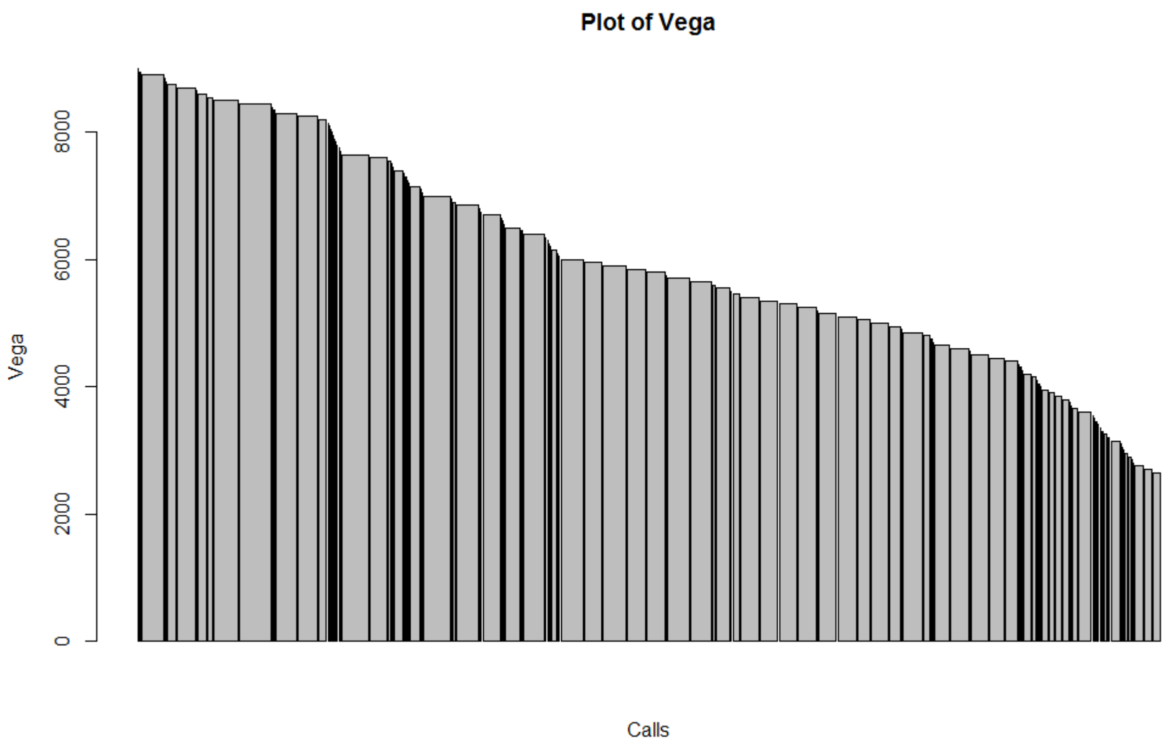
$$\frac{\partial^2 C}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{T-t}}$$



As shown from the graph above, the range of the gamma results ranging from (0, 8000) with changing prices which is the call price, C.

Vega

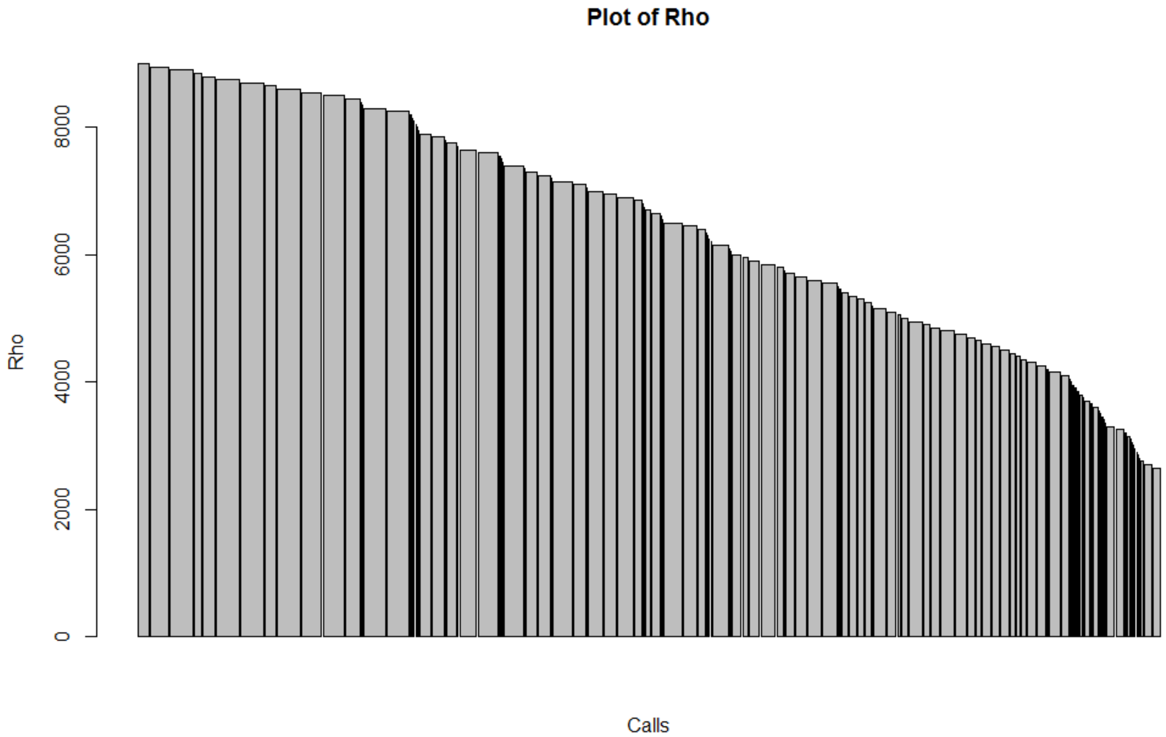
$$\frac{\partial C}{\partial \sigma} = SN'(d_1)\sqrt{T-t}$$



The Vega graph also has a range of values which range from (0,8000) with changing price, C.

Rho

$$\frac{\partial C}{\partial r} = K(T-t)e^{-r(T-t)}N(d_2)$$



The Rho also has a range of value ranging from (0,8000) with increasing change in premium price, C.

In conclusion,

From the above results, it can be seen that the Gamma, Vega and Rho graphs have the same range of values with respect to the changing prices and therefore the sensitivity of the parameters of r, s and standard deviation. The graphs also show that the sensitivity analysis has a decreasing trend with increasing premium price and therefore the model is valid for pricing insurance premiums.

CONCLUSIONS AND RECOMMENDATIONS

CONCLUSIONS

The major objective of this research is to price an insurance product for *'boda-boda'* operators. We can conclude that this objective has been achieved. This pricing can be done using both actuarial methods (EBCT I and II) and the proposed Black Scholes model. The following premiums are appropriate depending on the chosen model:

MODEL	PREMIUM

EBCT I	67,018
EBCT II	53,935
Black Scholes Model	54,928

It is clear that all models result in roughly the same premium. Therefore, the only consideration an insurer has to make in pricing such a product is the cost of implementing a model.

In addition, we can conclude that it is possible to translate the parameters of the Black Scholes model that was originally designed to price European call options, to fit an insurance product for *'boda-boda'* operators. With such translation done quite easily and application of the R software, we can conclude that Black Scholes model is an efficient and effective method of pricing the product investigated in our research.

The sensitivity analysis done on the proposed Black Scholes model revealed that the Greeks have inverse relationships with the calls as they are expected to. It was also observed that the parameter that is most sensitive in determining the premium is the value of the underlying S_t.

RECOMMENDATIONS

Based on the research we have conducted, we propose the following:

- The '*boda-boda'* insurance product should be priced using the translated Black Scholes model as this model is updated to deal with the changing trends in the insurance market and it allows for the sensitivity analysis which is vital in risk analysis.
- The insurers offering this product in the market and using the actuarial valuation methods (EBCT) should use the Black Scholes model to check their premium levels.
- The insurers who are not offering a '*boda-boda'* product should consider doing so since the models to price it are already freely available. The low premiums calculated show that such a product is attractive to the target market and thus the product will be an added stream of revenue to these insurers.

LIMITATIONS

As expected, limitations were encountered as seen in previous works of related studies[5]

The Black-Scholes model only assumed that the insurance cover acted for a single year. The results would have been different in the event when multiple years were assumed.

The data on the motorcycle operator was difficult to access and is there is limited research done on it.

Boda-boda Insurance cover is relatively unexplored. Therefore plenty of assumptions were made so as to arrive at the conclusions. Fewer assumptions would have resulted in a more accurate depiction of the product model.

[5] See page 6 *"Use of pure premium models in automobile insurance market"*.

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Appendix I: R-codes

Rho and plotting of Rho

```
#rho

Rho<-strike*exp(-r)*pnorm(d2)

Rho

rho<-mean(Rho)

rho

range(Rho)

plot(call,Rho,xlab="Calls",ylab="Rho",main="Plot of Rho")
```

Delta and plotting of Delta

```
#delta

Delta<-pnorm(d1)

Delta

delta<-mean(Delta)

delta

range(Delta)

plot(call,Delta,xlab="Calls",ylab="Delta",main="Plot of Delta")
```

Gamma and plotting of gamma

```
#gamma

Gamma<-dnorm(d1)/(call*sigma)

Gamma

gamma<-mean(Gamma)

gamma

range(Gamma)

plot(call,Gamma,xlab="Calls",ylab="Gamma",main="Plot of Gamma")
```

Vega and plotting of vega

```
#vega

Vega<-call*dnorm(d1)
```

```
Vega
vega<-mean(Vega)
vega
range(Vega)
plot(call,Vega,xlab="Calls",ylab="Vega",main="Plot of Vega")
```

Appendix II: Microsoft Excel Screenshots

Total claim amounts

	total claim amounts					
policy A	2701244	8987026	15545129	14734552	15984754	30979269
policy B	109081059	126716301	37007880	33370642	54069694	72646607

Calculation of the means.

23	policy	mean of x		
24	A	169350	1.94E+13	7.98E+14
25	B	671598.8	8.23E+14	1.38E+15
26	total	840948.8		

Calculation of the means.

23	policy	mean of x		
24	A	169350	1.94E+13	7.98E+14
25	B	671598.8	8.23E+14	1.38E+15
26	total	840948.8		

Calculation of Variations

604	YEAR	$\bar{y}_i$	s_i^2	$(\bar{y}_i-\bar{y})^2$
605	2010	31,922.10	2,874,038,781.16	25,982,550,080.21
606	2011	51,225.21	1,978,166,268.61	20,132,183,410.48
607	2012	53,974.81	2,761,725,466.98	19,359,474,609.44
608	2013	55,991.01	5,001,995,352.38	18,802,478,659.77
609	total	193,113.14	12,615,925,869.12	84,276,686,759.90

EBCT values

619	YEAR	EBCT
620	2010	31,925.19
621	2011	51,224.66
622	2012	53,983.90
623	2013	55,989.56
624	2014	67,018.92
625	2015	74,514.15

1		2010	2011	2012	2013	2014	current year
2	policy A	85	254	316	303	318	564
3	policy B	212	485	691	597	697	1077