UNDERWRITING RISK MANAGEMENT STRATEGIES FOR MOTORVEHICLE INSURANCE: A CASE OF THE KENYAN ALLIANCE INSURANCE COMPANY

BY

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DECLARATION

This research project is my original work and has not been submitted for a degree in any other University.

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To my dear husband Daniel Jumbale Kitti, my daughter Olive Mpenzwe Jumbale, my parents Samuel Ndeda and Mildred Ndeda and my sisters Rehema Ndeda, Rebecca Ndeda and Rowena Ndeda. I thank you all for this opportunity. May the Lord bless you abundantly.

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To God be the glory and honour, for the grace and mercies during the entire period of study. I thank you Lord for the gift of life and good health that you provided me over the two year study period.

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ABSTRACT

In recent years, risk management has been the subject of financial scrutiny due to the financial crises of 2002 and 2007. The Ukrainian and Greek case have also highlighted great weaknesses in financial risk management. Despite the existence of risk management policies and strategies, it is seen that implementation of these strategies is still an uphill task. The insurance industry, which is also part of the finance industry, has not been spared in terms of their existing risk management strategies.

The United Kingdom and India are seen as market leaders in the insurance industry. These market leaders have employed various risk management strategies. The United Kingdom uses the Solvency II model of risk management, which considers reserving, capital adequacy requirements and pricing of insurance products. India employs risk modeling among others as their risk management strategies.

The Kenyan insurance industry is unique in terms of dealing with underwriting risk. Rate pricing and rate reserving are used as opposed to risk related pricing and reserving strategies. Risk modeling is rarely used in computation of aggregate claims. The main objective of the study was to apply risk modeling as a risk management strategy in the Kenyan insurance industry.

The study relied on secondary data obtained from The Kenyan Alliance Insurance Company Limited. The data reviewed was motor vehicle claims data for 2010. Exploratory statistical tests were carried out before modelling the number of claims and the claims severity. Aggregate claims were then determined by use of the collective risk model. Expected premiums for year 2011 we obtained using the expected value principle, standard deviation principle and credibility premium.

The study found that the expected premiums were much higher than the rate computed premiums. The result suggests that for the same amount of claims that an underwriter has to pay, the premium charged should be higher.

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CHAPTER ONE

INTRODUCTION

1.1 Background of the study

1.1.1 An overview of the Insurance Industry

Insurance is a business of trust. It is a business whereby a promise is sold and such a promise must be kept. An insurance company offers insurance cover and is also known as the underwriter. The insured is the one who has taken an insurance policy. Three parties come to play in such an arrangement: the policyholders, shareholders and the management of the insurance company. Policyholders are the buyers of the insurance policies. They expect to be paid from the insurance fund when they make a claim. Shareholders are the owners of insurance companies. They expect to make profits from the insurance fund. The management is involved in the day to day running of the insurance company. They do this on behalf of the shareholders. The management of the insurance company owes fiduciary responsibility to the policyholders. They hold the policyholder's money in trust in order to indemnify them in case of a loss.

According to Pithadia (2006), insurance is a growing business in India. India's life insurance penetration is at 19% whereas most countries are grappling with 3% - 5% penetration. The banking and insurance sectors add about 7 percent to the country's Gross Domestic Product (GDP). The Indian insurance companies are governed by two acts of parliament: The Insurance Act (1938) and IRDA Act (1999). In India, insurance is generally considered a tax-saving device. The Life Insurance Corporation of India dominates the Indian Insurance Sector. Life insurance in India is said to have been started in 1818. Currently, there are twelve general insurance companies in India, four public and eight private.

Apra (2012) gave an insight into the Australian Insurance Industry. The main challenges in 2011/2012 year of income included hardening the property reinsurance market and large falls in interest rates. Despite this, the insurer's profitability and solvency remained strong. The Australian insurance industry had 124 authorized insurers and reinsurers; with 102 actively writing business and 22 in a run-off. Inadequate reserving is viewed as a great risk in relation to the general insurance industry. This is seen to expose the insurers to large losses if their claims experience deteriorates. The Australian life insurance industry faces specific concerns for example capital valuation adequacy in the face of future asset valuation shocks, data management, worsening claims experience, group life pricing standards and governance practices underpinning the rapid growth in directly marketed risk products.

According to the Insurance Industry Annual Report (2012), insurance industry penetration in Kenya has increased from 2.5 percent to 3.16 percent in the last seven years. This improvement in penetration is attributable to improved regulatory framework, innovative products, adoption of alternative distribution channels, enhanced public education and use of technology. Insurance performance relative to the GDP at market prices has been increasing since 2008. Generally, the financial services sector recorded a negative growth of 6.5 percent in 2012 as compared to 7.8 percent in 2011 due to the tight monetary policies in the first half of 2012.

The Kenyan insurance sector constitutes 46 operating insurance companies as at the end of 2012, 23 insurance companies writing non-life business, 11 writing life business only and 12 composite insurance companies. There are a total of 170 insurance brokers among other service providers. Non-life insurance otherwise known as general or short-term insurance covers property and casualty risks which in Kenya are divided into 15 broad classes. The classes are: Motor, Fire, Workmen's Injury Benefit, Personal Accident, Marine, Theft, Medical Insurance, Liability, Aviation, Engineering and miscellaneous.

Life Insurance is a long-term contract between the policy holder and the insurer that facilitates long term savings. In the event of death of the policyholder, the beneficiaries are paid the agreed sum assured. Other events in life that may trigger payment include: critical illness, terminal illness and temporary/ permanent disability. The broad classes of life insurance are ordinary life assurance, group life assurance, deposit administration scheme and investment or unit linked contracts. Ordinary life assurances comprise all individual life policies for example term assurance policies and whole life policies. Group life assurances comprises group life schemes mainly organized by employers on behalf of their employees for example group credit or group mortgage schemes which are loan protection schemes organized by financiers. Deposit administration or pension business is a unique class of life insurance business which gives a guarantee on the capital and a minimum rate of return on the pension funds. Investment or unit linked contracts has the main objective of facilitating the growth of capital invested by the client or insured.

The Association of Kenya Insurers (AKI) was established in 1987. Its main objectives are to enhance professionalism for effective management of the insurance industry, enhance underwriting standards and develop best practice guidelines, creation of positive image of the industry through public education, lobbying Government for the creation of an enabling legislative environment and research.

The Insurance Regulatory Authority (IRA) was started in May 2007 through an Act of Parliament as an autonomous Government Institution. IRA took up the functions of the former Department of Insurance and is run by the Commissioner of Insurance (COI). IRA is charged with regulating, supervising and developing the Kenyan insurance industry. Other roles of the IRA include: formulating and enforcing standards, issue licenses to all persons, prompt settlement of claims, investigate and prosecute all insurance fraud.

1.1.2 Risk Management in regards to the insurance industry

In a broad sense, a risk is the possibility of losing all or some of an original investment. A risk could also be defined as a probability or threat of damage, injury, liability, loss, or any other negative occurrence that is caused by external or internal vulnerabilities, and that may be avoided through pre-emptive action. In relation to finance, risk is the probability that an actual return on an investment will be lower than the expected return. There are various categories of financial risk for example exchange rate risk, interest rate risk, liquidity risk, political risk, reinvestment risk, solvency and underwriting risk among others.

A survey conducted by Everis in 2009, categorized the various risks experienced in the insurance industry as underwriting risks, credit risks, market risks, operational risks and liquidity risks among other risks. Underwriting risks were defined as the risks that occur when premiums are computed inaccurately, aggregate claims are incorrectly computed and claims reserves are understated. Understatement of claims reserves and incorrect computation of aggregate claims could result in future company losses. Inaccurate calculation of premiums has adverse financial implications to the company especially in relation to payments of claims. Credit risks are those risks which are associated to debtor default. Market risks are those that result from fluctuation of all relevant market prices. Operational risks are those that arise from inadequate organizational internal processes whereas liquidity risk denotes risk associated with the financial market.

Risk may be avoided through pre-emptive action which involves risk management. According to Njuguna and Arunga (2012), there are various risk management strategies available to micro-insurance companies. These include: efficient distribution channels of insurance products, price reviews, scrutiny of micro-insurance claims and applicants, introduction of flexible premium payment plans, niche marketing and responsive regulation among others.

Dowd et al (2007) defines risk management strategies for the financial industry to constitute clearly set out risk policies, an independent risk management function headed by a Chief Risk Officer, risk modeling and timely communication of risk matters. The risk management function would be responsible for formulation and implementation of risk control systems. Risk modeling involves the use of models that estimate risk measures and possibly carry out stress tests.

1.1.3 Underwriting risk in the Kenyan context

As seen earlier, underwriting risk stems from inaccurate premium computation, incorrect modelling of aggregate claims and inadequate reserves set aside by the insurance companies. Between 2009 and 2014, most short term underwriters have been victims of inadequate reserves due to hefty court awards given for certain claims. An example is where limbs were lost, the courts had previously awarded an amount of Kes 20,000. This has now been revised upwards to an amount of Kes 300,000. Ideally, an underwriter who had reserved funds based on Kes 20,000 would have been a victim of underwriting risk.

A second risk factor would be in the lack of risk modelling of aggregate claims. Modelling of aggregate claims based on the past claims enables an insurance company to reserve for claims and to price for premiums. Lack of risk modelling of the past claims exposes a firm to insurance risk. A large number of insurers have not done a risk model for their aggregate claims. However, a good number of insurers affirm that claims is the riskiest area in running an insurance company especially those relating to motor vehicle claims.

Another risk factor is the use of rate calculation of premiums especially in motor vehicle insurance. In recent years, underwriters have settled on a rate of 3-4% of the motor vehicle value; to be the premium charged for insuring vehicles. The Commissioner of Insurance sets these rates as per the Insurance Act, however, cut-throat competition pushes the determined rates downwards. Therefore, chances of suffering risks relating to underwriting risk are very high in Kenya and appropriate strategies need to be put in place.

1.2 Statement of the problem

This study has been necessitated by the varying risk management strategies that exist worldwide and the lack thereon of an appropriate underwriting risk management strategy in Kenya. Due to the financial scandals and market failures that were experienced between 2002 and 2007, risk management keeps regaining its relevance. In fact, it is believed that despite having the best risk management strategies, the implementation and actualization of such strategies has not been concentrated on.

The United Kingdom adopted the Solvency II model which was derived from the Basel II and Basel III banking accord. The risk based requirements of the solvency II model include technical provisions in the balance sheet and minimum capital requirements, among others. Technical provisions include coming up with well modelled reserves in relations to claims paid and claims outstanding. The importance of the minimum capital requirements is to make sure that for every risk that an insurer takes, there is an adequate capital reserve that ensures solvency.

As at 2011, the United States insurers had not adopted any regulations in relation to stochastic reserving and capital adequacy requirements. These insurers were considering the adoption of Solvency II earlier in 2012. The American and United Kingdom insurers pay a lot of attention to risk management because of the ratings that their companies receive. The recent financial scandals have necessitated a keen eye on the risk management companies.

The Kenyan insurance industry reveals the lack of a risk management strategy and hence the main object of this study. Risk management strategies that relate to underwriting risk have not been employed in Kenya. Premium calculation and claims reserving have been done by use of a specified rate as per the directives of the Commissioner of Insurance in The Insurance Act, CAP 487 (Laws of Kenya). However, use of rate method has been seen to undervalue the claims paid and results in very high risk taken as compared to the solvency of the company. This in turn results in a company that is not able to honour the insurance promises and hence the collapse of such a company.

1.3 Objectives of the study

1.3.1 Main Objective

i) To employ claims risk modelling as a risk management strategy for the motor vehicle class of business.

1.3.2 Secondary Objectives

ii) To determine the credibility premium chargeable for every event of loss and in turn provide an alternative to the rate pricing technique.

iii) To study Kenyans risk management strategy vis a vis the strategies that are used world-wide.

iv) To understand the insurance operations of other jurisdictions.

v) To provide viable solutions to the risk management questions in the Kenyan insurance industry.

1.4 Significance of the Study

This study is important to the Insurance Regulatory Authority as it shows the benefit of risk management in underwriting risk as compared to the rate method. The study attempts to curb the challenge of under-reserving that has been experienced in the recent past in the insurance industry. It enhances adequate reserving measures such that the claims can be paid whilst insurance companies adhere to the solvency requirements.

Adequately priced insurance products will also benefit the insurance industry in terms of unwarranted and unhealthy competition. The insurance companies will also be able to cover risks of which they can be able to afford and pay in case of a claim. This should be done without the company going under.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter reviews the past literature on risk management and risk management strategies. The theoretical and empirical review is also critically analyzed. In addition to these, the chapter reviews the past and current Kenyan insurance law with specific interest in risk management and risk management strategies.

2.2 Empirical Review

2.2.1 An overview of International Insurance companies

Various studies on risk management and prevention of such risk have been done. These studies date to as far back as World War Two. Harrington and Neihaus (2003) date the origin of risk management to 1955-1964. They further associate risk management with the use of market insurance to protect individuals and companies from various risks associated with accidents.

A brief history of risk management was given in Georges (2013), which highlighted the emergence of pure risk management as an alternative to market insurance in mid 1950s. 1970s and 1980s saw the development and use of derivative instruments which were further discredited due to their risky and ambiguous nature. Georges further demonstrates that despite the development of financial risk models and capital calculation formulas by the financial institutions, financial crises of 2002 and 2007 are inevitable. He pre-empts that this could be as a result of lack of implementation of risk management strategies.

Dionne (2009) isolates the major risk management problems as lack of incentive contracts in the presence of informational asymmetry, poor valuation of structured products by rating agencies, poor pricing of complex financial products and poor regulation of structured finance. Georges (2013) concludes by saying that risk management includes minimizing the company’s risk as well as maximizing the firm’s value. Companies can use internal activities to protect themselves from risk; however, the risk positions of financial institutions expose their customers to great risk despite the institutions intending to increase their return.

A survey conducted by Everis in 2009, on the risk management in the insurance industry in Europe and South America drew various conclusions. In Spain, 73% of the companies under consideration had a reserve allocated to risk management, 18% did not have such a reserve and the remaining 9% were not even intending to put up such a reserve. The risks that had been identified include deviation risks, insufficient premium risk, technical reserve risk, reinsurance risk, major losses risk, liquidity risk, general business risk and operational risk among others.

Portugal was considered more advanced in terms of their risk management. 90% of the Portuguese companies under consideration dealt with their own risk as a separate unit within the organization, the remaining 10% had specific individuals tasked to handle the affairs of risk within the organization.

Brazilian companies were also considered to be more advanced in regards to risk management as they were well aware of all the existent risks. It was interesting to note that they considered that the main reasons hindering efficient risk control was lack of systems evolution and difficulty in implementation of methodology in the companies, a conclusion shared by Blanchard and Dionne (2003, 2004) in regards to the 2002 New York Stock Exchange Financial crisis. The methods used in risk calculation by the Brazilian companies include stress testing, the mix method, the parametric and the deterministic methods.

According to Jason Thacker (2011), the European insurance industry adopted the Solvency II risk management model, which was developed from the Basel II and Basel III framework of the Banking Industry. The risk based requirements of the Solvency II model include technical provisions in the balance sheet and minimum capital requirements, among others. The solvency II model has been faulted to reduce foreign insurance and long-tailed business exposure and hence a shift of foreign business to United States (US).

As at 2011, the US insurers were yet to adopt regulations concerning stochastic reserving and capital adequacy requirements. Over the years, increased attention has been laid on financial risk management. Actuaries have been faulted for reliance on deterministic measures of risk rather than adopting stochastic measures. Actuaries have also focused more on measureable and frequent risk events which are difficult to do with measuring of operational risk.

The Indian market has chosen to uniquely deal with risk management and some of their strategies employed include conducting pre-and post-risk audits, risk modeling using various catmodels by mathematical simulation carried out to handle catastrophic risks and logistic risk management by use of data loggers. They also engage a lot of risk management consultants. Sethi (2007) says that the Mumbai flood led into the employment of more risk management strategies. Flood modelling and other catastrophic modeling are being developed to quantify catastrophes and it is becoming a tool for pricing insurance and reinsurance products.

2.2.2 Risk perception in relation to risk management.

Risk management and risk perception go hand in hand. Chieng-Chung et al (2013) attempted to connect risk culture theory, risk perception with underwriting performance of underwriters in property or liability insurance companies and create a linkage between risk perception and decisions of risk management strategy. Risk perception was found to represent a considerable part of the underwriting process. The underwriting performance and underwriter's worldviews did not reveal a visible pattern. This is because underwriting performance is a biased estimate and reflects individual performance.

2.2.3 The Kenyan case

Non-life insurance in Kenya adopted a rate approach to risk management. Chapter 487, Laws of Kenya, Section 76, states how the premiums of different classes of non-life insurance are arrived at. The proposed manual of rates or rates schedule for every class of insurance is submitted to the Commissioner of Insurance by the insurance company. The Commissioner then approves these rates depending on his discretion and the industry at large.

Through the years, it has been noted that there exists cut-throat competition in the insurance industry. In specific, motor vehicle comprehensive insurance rates have been changing over the years. In 2007, the rate employed on motor vehicle comprehensive insurance was 7.5% of the motor vehicle value, whereas in 2014, this rate has gone as low as 3.5% of the motor vehicle value. In contrast, the motor vehicle third party insurance rates have been constant at a rate of Kes 2,500 per month for a small vehicle and Kes 7,500 per year for the same motor vehicle. Premium pricing especially for the motor vehicle class of insurance is not risk based and hence not in tandem with insurable risk for example expected claims.

In relation to claims reserving, CAP 487, Section 54 (2) provides that "*Every reserve shall be calculated in accordance with the method approved for the purpose by the Commissioner".* In practice, these reserves are calculated as a percentage of the claims outstanding which is 15%. Such reserves are created to meet future obligations as said in Section 4A (3) (b). CAP 487, also provides for actuarial intervention on long term business for carrying out valuation. It is arguable that actuarial intervention in the case of uncertainty would be prudent. However, the Act as is sees this as unnecessary.

In 2013, the Insurance Act, CAP 487, Laws of Kenya was repealed and a draft bill for 2014 is available. Full implementation of the draft bill is yet to be achieved as it has not been signed into law by the President. An overview of the draft bill shows that the Commissioner of Insurance is turning from rate pricing and reserving to adopting risk management strategies. In specific, Section 55 of the 2014 Insurance draft bill is on risk management.

Section 55 (1) of the draft Insurance Bill says that, "*A licensed insurer shall establish and maintain:-*

> *(a) a clearly defined strategy and policies, for the effective management of all significant risks that the insurer is or may be exposed to; and*
>
> *(b) procedures and controls that are sufficient to ensure that the risk management strategy and policies are effectively implemented.*

Section 55 (2) of the draft Insurance Bill says that, "*The risk management strategy and policies shall:*

- *(a) Be appropriate for the nature, scale and complexity of the licensed insurer's business;*
- *(b) Specify how risks are to be identified, monitored, managed and reported on in a timely manner;*
- *(c) Take into account the probability, potential impact and the time duration of risk; and*
- *(d) Comply with such other requirements as may be specified in the Regulations.*

Section 55 (3) of the draft Insurance Bill says that, *" Without limiting subsections (1) and (2), a licensed insurer's risk management strategy and policies shall provide for—*

> *(a) insurance risk;*
>
> *(b) counterparty default risk;*

(c) liquidity risk;

(d) operational risk;

(e) balance sheet and market risk;

(f) strategic risk and investment risk;

(g) contagion and related party risk;

(h) legal and regulatory risk; and

(i) such other risks as may be specified in the Regulations

Section 56 of the draft Insurance bill 2014 provides for an actuarial function, risk management function and compliance function amongst other functions in a licensed insurance organization. In summary, the proposed law recognizes the value of risk management in insurance companies. The proposed law also provides that the insurers acknowledge the various risk types that exist and provide related strategies that could handle such risk.

Barth and Feldhaus (1999) examined the effect of country rate regulation on the profitability and stability of individual insurers in the private passenger auto insurance market in America. The specific reasons for selecting auto insurance was because there was readily available quantitative data and the coverage was large. The study concluded that underwriting risk was not any different in the presence of prior-approval rate regulation. Overall regulatory climate was found to have some relationship to underwriting risk experience by insurers in each of the examined country's market. This study goes to show the importance of risk management as rate regulation is seen to result in underwriting risk.

2.3 Theoretical Framework

2.3.1 Financial Economics Theory

Financial Economics approach to financial risk management builds upon the classic Modigliani and Miller (1958) which states the conditions for irrelevance of financial structure. This was later extended to risk management which stipulates that hedging leads to lower volatility of cashflows and therefore lower volatility of the firm value. Hedging is a form of risk management which was preferred in the 1970s and 1980s through derivatives Georges (2013). The ultimate result of hedging was indeed beneficial to the firm and resulted in a higher value – a hedging premium. Evidence to support the predictions of financial economics theory approach to risk management is poor. However, according to Jin and Jorion (2006), risk management does lead to lower variability of corporate value.

Harrington (1984) proposed three theories on the effects of rate regulation on the profits of insurance companies. These are: regulatory-lag hypothesis, excessive rate hypothesis and the consumer-pressure hypothesis.

2.3.2 Regulatory – lag hypothesis

The regulatory – lag hypothesis theory states that the regulatory delays in approving rate filings result in delays for companies trying to implement new rates and react to market changes. As mentioned above, CAP 487 requires every insurer to submit to the Commissioner of Insurance the prescribed premium rates schedule for every class of insurance. Such submission would result to delays in reaction to any market changes. Implementation of further market changes would demand another submission of the premium rates to the Commissioner of Insurance. The theory continues to say that in the long-run, there are no differences in profit ratios between rate regulated and unregulated jurisdictions. However, in the short run, rate regulation provides a somewhat cyclic behaviour. This behaviour leads to more variability in underwriting results in the rate regulated states thus higher risk. If this theory holds true, long-run underwriting profits should not be affected by the rate regulation. In the short –run, underwriting results may be affected by rate regulation. In the long-run the market evens out and hence profits are largely unaffected by the rate regulation.

2.3.3 Excessive Rate Hypothesis.

The excessive rate hypothesis assumes that the regulators protect consumers against insolvency risk through minimum rate floors that reduce cutthroat competition. As observed in the insurance industry here in Kenya, cutthroat competition is rife. Insurers are willing to give even the lowest price in order to retain a client. At times the insurers may use very low prices that make no business sense but assist in building up clientele. Price floors have previously been set in the Kenyan market e.g. 7.5% of the value of the motor vehicle. Despite this, the Kenyan insurers have been seen to go below this floor and not above it as expected by the theory. The intention to protect end users of insurance by the regulator is altered by the non-adherence to the price floors.

The excessive rate hypothesis fairly means that on average, insurers are forced to charge rates that are higher than they would otherwise be in the absence of rate regulation. If the hypothesis holds true, the average profit ratio in the rate regulated states would be higher than the average profit ratio in open-competition states. The conclusion of this theory may be arguable especially because an insurer who does not adhere to the price floor may still make higher profits as they may have a larger clientele base.

2.3.4 Consumer-Pressure Hypothesis

The consumer-pressure hypothesis states that consumers pressure the regulators to restrict prices to enhance affordability, so prices would be below the level that the competitive market would establish in rate regulated states. Ideally, the regulator can restrict such prices if pressured or compelled to do so. In case consumer pressure works, the average profit will be lower in the rate regulated countries.

2.3.5 The culture theory of risk

This theory attempts to explain risk perception. Mary Douglas (1982) explains the four attitudes to risk:

i) Individualists: They are self-correcting, similar to mean reverting and not concerned about risk. Individualists believe in an unbounded growth system.

ii) Egalitarians: They believe that a major change could result in a disaster thus have finite resources to curb disaster. Egalitarians believe in being accountable.

iii) Authoritarians: Risk taking is acceptable if controlled by experts. They have rules and laws to keep risk taking under control. They have a high degree of concern for consequence.

iv) Fatalists: They believe that the world is uncontrollable and unpredicatable.

Ingram and Thompson (2010) gave a new perspective to the four attitudes towards risk. They thought of the attitudes of risk as different business strategies.

i) Individualists were thought to be profit maximisers. They focus on return and not risk.

ii) Egalitarians were thought of as conservators who are concerned with risk and avoid risk at all costs. They exhibit a traditional form of risk management which is loss controlling strategy.

iii) Authoritarians are viewed as risk-reward managers. They use risk steering when the risk environment is moderate

iv) Fatalists are pragmatic and favour diversification to mitigate risk.

2.4 Summary of the Literature Review.

Risk management in the insurance industry is a dynamic issue. As years go by, the risk management strategies need to be enhanced. United States adopted the Solvency II model of risk management in 2012. The United Kingdom had already adopted the Solvency II model. Actuaries have also been faulted for relying on the deterministic models as opposed to stochastic models.

Closer home, the risk management strategies have not been dwelt on in the insurance industry. This is a new concept and viewed with great resistance by the insurers. It would be interesting to see the uptake and effectiveness of risk management in Kenya. Risk management requirements may be stringent and inapplicable with teething problems at inception as seen in Thacker (2011).

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

This chapter sets out the mathematical model that was used in the analysis of data and its application. Various derivations as well as theorems are proved.

3.2 Aggregate Loss Models

An aggregate loss is similar to an aggregate payment. This is the total amount paid or the total losses on all claims occurring in a fixed period on a defined set of insurance contracts. There are two standard situations for modeling aggregate losses: when there is collective risk and when there is individual risk.

Aggregate loss models inform us and allow us to make decisions on expected profits, premium loadings, reserves necessary to ensure a high probability of profitability and the impact of reinsurance and deductibles.

Definition 3.2.1: Collective Risk Model

A collective risk model represents the aggregate loss as a sum, S, of a random number, N, of individual payment amounts $\left(X_1, \ldots\ldots, X_N\right)$. Therefore

$$S = \sum_{i=1}^{N} X_i$$

It is assumed that all the random variables are independent and that X_i's have identical probability distribution. The most common independence assumptions are of the form:

i. Conditional on N=n, the random variables $X_1, \ldots., X_N$ are identical and independently distributed (iid) random variables.

ii. Conditional on N=n, the common distribution of the random variables $X_1, \ldots., X_N$ does not depend on n.

iii. The distribution of N does not depend in any way on the value of $X_1, \ldots., X_N$

Generally, under the collective risk model, S is a compound distribution and the model is compounded by the fact that the number of terms in the sum is random and not fixed. The term S could otherwise be referred to as the aggregate loss random variable or the total loss random variable.

Definition 3.2.2: Individual Risk Model

An individual risk model represents the aggregate loss as the sum of the amounts paid on each component of the portfolio of risks. Therefore

$$S = \sum_{j=0}^{n} Y_j$$

Y_j is the amount paid on the jth contract and it is assumed that $Y_1, ..., Y_n$ are independent. It is not assumed that the Y_j's have identical distributions. Every contract produces losses according to its own provisions and the underwriting characteristics of the policyholder.

The assumptions of the Individual Risk Model are that at most, one claim may arise from a policy whereas in the Collective Risk Model multiple claims may arise from a single policy or policyholder. For this project, the Collective Risk Model was used as there are multiple claims that arise from a single policy holder.

Definition 3.2.3: Claims count random variable

This refers to the number of claims, N, as a random variable which when modeled is referred to as claims count distribution. Another term that could be used is the frequency distribution. The number of claims is a discrete random variable.

Definition 3.2.4: Claims severity distribution

Severities refer to the individual or single loss random variables, X_i's and the Y_j's, which when modeled become the claims severity distribution. Another frequently used term is the loss. An assumption is made here as to the claims severity being a continuous random variable. The claims severity and claims count were modelled separately.

Modeling the severity and the frequency has very distinct advantages:

i. The expected number of claims changes as the number of insured policies change. Growth in business volume needs to be accounted for in forecasting the number of claims in future years based on past years' data.

ii. The effects of general economic inflation and additional claims inflation are reflected in the losses incurred by insurance parties and the claims paid by insurance companies.

iii. It is easier to study the effects of changing individual deductibles and policy limits for example reinsurance.

iv. It is easier to understand the effects of changing deductibles on claims frequencies.

v. Models which are developed for non-covered losses to policyholders, claim costs to insurers and the claim costs to reinsurers can be mutually consistent. This is useful for a direct insurer who wants to study the consequence of shifting losses to a reinsurer.

vi. The shape of the distribution of S depends on the shape of the distribution of N and X. Understanding these distributions become important for modifying policy details.

Definition 3.2.5: Compound Distribution

The random sum $S = \sum_{i=0}^{n} X_i$ (where N has a counting distribution) has the distribution function:

$$F_S(x) = \Pr ob(S \leq x)$$

$$F_S(x) = \sum_{n=0}^{\infty} p_n * \Pr ob(S \leq x \mid N = n)$$

$$F_s(x) = \sum_{n=0}^{\infty} p_n * F_X^{*n}(x)$$

$F_X(x) = \Pr ob(X \leq x)$ is the common distribution function of the X_i's and $p_n = \Pr ob(N = n)$

$F_X^{*n}(x)$ is the “n-fold convolution” of the cumulative density function (CDF) of X. It can be obtained as shown below:

$$F_X^{*0}(x) = \{\begin{matrix} 0 & x < 0 \\ 1 & x \geq 0 \end{matrix};$$

$$F_X^{*k}(x) = \int_{-\infty}^{\infty} F_X^{*(k-1)}(x-y) * dF_X(y)$$

The distribution $F_s(x) = \sum_{n=0}^{\infty} p_n * F_X^{*n}(x)$ is called a compound distribution and the probability function (pf) for the distribution of aggregate losses is given below as:

$$f_s(x) = \sum_{n=0}^{\infty} p_n * f_X^{*n}(x)$$

Proposition 3.2.1: Properties of Compound Distribution

i. The expectation of S is given by multiplying the expectation of the claims severity and the claims frequency.

$$E(S) = E(N) * E(X)$$

Proof of proposition 3.2.1 (i)

Using the property $E(S) = E_N(E(S \mid N))$ then:

$$E(S) = \sum_{n=0}^{\infty} E(X_1 + X_2 + ... + X_N \mid N = n) * \Pr ob(N = n)$$

Using the independence and identically distribution principle; $X_1 = X_2 = ... = X_N$

$$E(S) = \sum_{n=0}^{\infty} E(X + X + ... + X) * \Pr ob(N = n)$$

$$E(S) = \sum_{n=0}^{\infty} n * E(X) * \Pr ob(N = n)$$

$$= E(X) * \sum_{n=0}^{\infty} n * \Pr ob(N = n)$$

Consider that $E(N)=\sum_{n=0}^{\infty} n*\Pr ob(N=n)$ therefore $E(S)=E(N)*E(X)$ as earlier stated.

ii. The variance of the aggregate loss is given as shown below:

$$Var(S)=\left(E(X)\right)^2 *Var(N)+Var(X)*E_N(N)$$

Proof of proposition 3.2.1(ii)

Using the principle $Var(Y)=Var[E(Y\mid X)]+E[Var(Y\mid X)]$, the variance of the aggregate loss becomes;

$$Var(S)=Var[E(S\mid N)]+E[Var(S\mid N)]$$

From above $E(S\mid N)=N*E(X)$ and $Var(S\mid N)=N*Var(X)$ thus:

$$Var(S)=Var[N*E(X)]+E[N*Var(X)];$$

$$Var(S)=\left(E(X)\right)^2 *Var(N)+Var(X)*E_N(N)$$

iii. The probability generating function (pgf) of S is $P_S(z)=E\left[Z^S\right]$

Proof of Proposition 3.2.1 (iii)

$$P_S(z)=\sum_{n=0}^{\infty} E\left[Z^{X_1+X_2+......X_n}\mid N=n\right]*\Pr ob(N=n)$$

$$P_S(z)=\sum_{n=0}^{\infty} E\left[\prod_{i=1}^{n} Z^{X_i}\right]*\Pr ob(N=n)$$

$$=\sum_{n=0}^{\infty}\left[P_X(z)\right]^n *\Pr ob(N=n)$$

$$P_S(z)=E\left[P_X(z)\right]^n=P_N\left[P_X(z)\right]$$

The result above assumes independence of the X_i's and a fixed n.

iv. The moment generating function of S is given as $M_S(z)=E(e^{zS})=E_N(E(e^{zS}\mid N))$

Proof of Proposition 3.2.1 (iv)

$$M_S(z) = \sum_{n=0}^{\infty} E(e^{z(X_1+X_2+......X_N)} \mid N=n) * \Pr ob(N=n)$$

Due to independence and the X_i's being identical, we get;

$$= \sum_{n=0}^{\infty} E\left\{\prod_{i=0}^{\infty} e^{zX_i}\right\} * \Pr ob\left(N=n\right)$$

$$= E_N \left\{E\left(e^{zX}\right)\right\}^n$$

$$= E_N \left\{M_X(z)\right\}^n$$

$$= P_N \left[M_X(z)\right]$$

Alternatively, $M_s(z) = E_N\left(e^{n \log M_X(z)}\right) = M_N\left(\log M_X(z)\right)$

v. The characteristic function is given as $\varphi_S(z) = E\left(e^{izS}\right) = E\left\{E\left(e^{izS} \mid N=n\right)\right\}$

Proof of Proposition 3.2.1 (v)

$$= E_N \left\{E\left(e^{iz\left(X_1+X_2+.....+X_N\right)} \mid N=n\right)\right\}$$

Due to independence, we multiply the X_i's

$$= E_N \left\{E(e^{izX_1}) * E(e^{izX_2}) * .....E(e^{izX_N})\right\}$$

And due to the identical property of the X_i's;

$$= E_N \left\{E(e^{izX})\right\}^n$$

$$= E_N \left\{\varphi_X(z)\right\}^n$$

$$= P_N \left\{\varphi_X(z)\right\}^n$$

vi. The Laplace transform is given as $L_S(z) = E\left(e^{-zS}\right) = E\left\{E\left(e^{-zS} \mid N=n\right)\right\}$

Proof of Proposition 3.2.1 (vi)

$$L_S(z) = E\left(e^{-zS}\right) = E\left\{E\left(e^{-zS} \mid N = n\right)\right\}$$

$$= \sum_{n=0}^{\infty} E\left[e^{-zX_1+....-zX_N} \mid N = n\right] * \Pr ob(N = n)$$

$$= \sum_{n=0}^{\infty} E\left[\prod_{i=0}^{n} e^{-zX_i}\right] * \Pr ob(N = n)$$

$$= \sum_{n=0}^{\infty} \Pr ob(N = n) * \left[L_X(z)\right]^n = P_N\left[L_X(z)\right]^n$$

3.3 Computing the aggregate claims distribution

3.3.1 Introduction

There are four methods of computing the aggregate claims distribution. They are:

i. Approximating Distribution: This is whereby the methods of moments is used in parameter estimation of the approximating distribution.

ii. The recursive methods for example the use of Panjer's recursive formulae

iii. Inversion methods which include the Fast Fourier Method.

iv. Simulation method for example the Monte Carlo Simulation.

These four methods are numerical evaluation approaches.

Studies have been done to establish a more favourable method amongst the three. Embrechts and Frei (2010) compared the Panjer Recursion versus the First Fourier Transform (FFT) for the compound distribution. They concluded that Panjer's recursion is the most widely used method in evaluating the compound distributions. The First Fourier Transform is a viable alternative to Panjer's Recursion. FFT can be well applied with arbitrary frequencies. This study will apply the Panjer's Recursive Formulae.

3.3.2 Recursive Method

3.3.2.1 Panjer's Recursion Formulae

If the number of claims, N, in the collective risk model $S = X_1 + X_2 + ... + X_N$, has a Poisson distribution, and the claim random variables X_i's, takes positive integer values, then we may establish an exact recursive expression for the $\Pr ob(S = r)$ in terms of the probabilities; $\Pr ob(S = j)$ for j=0, 1, …., r-1 and the distribution of X.

Assume that N is a random variable with the recursive property that for some constants α and β, then;

$$\Pr ob(N = n) = \left\{ \alpha + \frac{\beta}{n} \right\} * \Pr ob(N = n-1)$$

Holds for n=1,…,max (N)

Suppose that the severity distribution $f_X(x)$ is defined on 0, 1,…., r representing multiples of some convenient monetary unit. The number r represents the largest possible payment and could be infinite. Suppose that the frequency distribution, p_k, is a member of the (a,b,1) class and therefore satisfies:

$$p_k = \left(a + \frac{b}{k} \right) * p_{k-1}; \text{ for k=2, 3, 4, …..}$$

Theorem 3.1:
For the model described above:

$$f_S(x) = \frac{[p_1 - (a+b)p_0] f_X(x) + \sum_{y=1}^{x \wedge r} \left(a + \frac{by}{x} \right) f_x(y) f_S(x-y)}{1 - af_X(0)}$$

Proof of Theorem 3.1

We begin from $p_k = \left(a + \frac{b}{k} \right) * p_{k-1}$ and multiply all sides by k to get $kp_k = (ka + b) * p_{k-1}$. We then introduce +a and –a to get: $kp_k = (ka) * p_{k-1} + (b) * p_{k-1} + a * p_{k-1} - a * p_{k-1}$. This in turn is simplified to give the following result:

$$kp_k = a*(k-1)*p_{k-1} + (a+b)*p_{k-1} \text{..................................(1)}$$

The result in (1) above is multiplied by $\sum_{k=1}^{\infty}\left[P_2(z)\right]^{k-1} * P_2^{'}(z)$ to yield:

$$\sum_{k=1}^{\infty} *k*p_k\left[P_2(z)\right]^{k-1} * P_2^{'}(z) = a\sum_{k=1}^{\infty} *(k-1)*p_{k-1}*\left[P_2(z)\right]^{k-1} * P_2^{'}(z) + (a+b)\sum_{k=1}^{\infty} p_{k-1}\left[P_2(z)\right]^{k-1} * P_2^{'}(z)$$

But $P(z) = \sum_{k=0}^{\infty} p_k * \left[P_2(z)\right]^k$ and $P^{'}(z) = \sum_{k=0}^{\infty} k*p_k*\left[P_2(z)\right]^{k-1} * \left[P_2^{'}(z)\right]$ we simplify the equation above to give us the result below:

$$P^{'}(z) = aP(z)\left[P_2(z)\right] + (a+b)P(z)P_2^{'}(z) \text{........................(2)}$$

$P(z)$ denotes a probability generating function whereas $P^{'}(z)$ is the first derivative of the probability generating function.

$$P(z) = \sum_{k=0}^{\infty} \Pr ob(S=k)*z^k$$

$$P^{'}(z) = \sum_{k=0}^{\infty} k*\Pr ob(S=k)*z^{k-1}$$

Then equation number (2) can be expounded in powers of z. The coefficients of z^{k-1} in such an expansion must be the same on both sides of the equation. Hence for k=1,2,3,…. We have:

$$kg_k = a\sum_{j=0}^{k}(k-j)f_j g_{k-j} + (a+b)\sum_{j=0}^{k} jf_j g_{k-j}$$

$$= akf_0 g_k + a\sum_{j=1}^{k}(k-j)f_j g_{k-j} + (a+b)\sum_{j=1}^{k} jf_j g_{k-j}$$

$$= akf_0 g_k + a\sum_{j=1}^{k} kf_j g_{k-j} + b\sum_{j=1}^{k} jf_j g_{k-j}$$

Therefore,

$$g_k = af_0 g_k + \sum_{j=1}^{k}\left(a + \frac{bj}{k}\right) f_j g_{k-j}$$

Rearrangement of this yields:

$$g_k = \left(\frac{1}{1-af_0}\right)\sum_{j=1}^{k}\left(a + \frac{bj}{k}\right) f_j g_{k-j}$$

Let g_k be the probability that S claims are equal to k, and further let this be represented as $f_S(x)$; let f_j be the probability that the claims X are equal to j and let this be represented as $f_X(y)$; Let g_{k-j} be the probability that the claims, S, are equal to k-j and that they are represented by $f_S(x-y)$. Also let j be equal to k, we get:

$$f_S(x) = \frac{\left[p_1 - (a+b)p_0\right] f_X(x) + \sum_{y=1}^{x\wedge r}\left(a + \frac{by}{x}\right) f_x(y) f_S(x-y)}{1 - af_X(0)}$$

Corollary 3.1

For the (a,b,0) class, the result above reduces to;

$$f_S(x) = \frac{\sum_{y=1}^{x\wedge r}\left(a + \frac{by}{x}\right) f_x(y) f_S(x-y)}{1 - af_X(0)}$$

When the claims severity distribution has no probability at zero, then $p_1 - \left(a+b\right) p_0 = 1$. In the case of the Poisson distribution with parameter (λ); then the distribution becomes:

$$f_S(x) = \frac{\lambda}{x}\sum_{y=1}^{x\wedge r} y f_x(y) f_S(x-y) \text{ for x=1,2,3….}$$

And specifically for the Poisson Distribution $f_S(0) = e^{-\lambda[1-f_X(0)]}$

3.4 Computation of Premiums for aggregate claims

3.4.1 Introduction

After analyzing the random risk, S, an insurance company would want to decide on how much they should charge to handle the risk and whether they should set aside reserves in the case of extreme or unlikely events.

3.4.2 Determining premiums for aggregate claims

Given a risk, S, we refer to its expected value E(S) as the pure or office premium for the risk. Insurance companies must charge more than the pure or office premium to cover their expenses, allow for variability in the number and amount of claims and also make a profit. The net premium is obtained by making an allowance for security or safety in determining a premium for a risk. In addition to these, the gross premium takes into account the administrative costs.

An assumption is made of nil administrative costs and thus a concentration of pure and net premiums. The model used for this computation assumes a loading of θ, whereby the net premium charged is of the form $(1+\theta)E(S)$. A large value of θ will give more security and profits but also could result in a decrease in the amount of business done because of the competitive nature of the insurance business. This principle of premium calculation is referred to as the expected value principle.

The standard deviation principle is also used for premium calculation. This method caters for the variability of the risk S. The standard deviation premium is based on $E(S)+\theta\sqrt{Var(S)}$. The variance principle is used to calculate the premium and it caters for the variability of risk. It is based on $E(S)+\theta Var(S)$.

3.4.3 Determining of the credibility premiums

Credibility theory is a set of quantitative tools that allow an insurer to perform prospective experience rating (adjust future premiums based on past experience) on a risk or group of risks. Generally, the rate pricing is designed to reflect the experience of the entire rating class and implicitly assumes that the risks are homogeneous. Credibility is motivated by various considerations:

i. The more past information the insurer has on a given policyholder, the more credible the policyholder's own experience.

ii. Competition may force insurers to give the policyholders' full credibility in order to retain the business.

iii. Other classes of insurance may lack actual past experience and thus application of credibility may be difficult.

Partial credibility premium is determined through the weighted average:

$$P_C = Z\overline{X} + (1-Z)\mu$$

Z denotes the credibility factor and this lies between 0 and 1. Z is determined in line with the actuarial techniques shown below:

$$Z = \frac{n}{n+k}$$

μ denotes the mean which is given by $E(X_j)$ and the variance, $Var(X_j)$ is given below:

$$Var(X_j) = E\left[Var\left(X_j \mid \theta\right)\right] + Var\left[E\left(X_j \mid \theta\right)\right]$$

$$= E\left[v\left(\theta\right)\right] + Var\left[\mu\left(\theta\right)\right]$$
$$= v + a$$

$$Cov(X_i, X_j) = a$$

$$k = \frac{v}{a}$$

CHAPTER FOUR

DATA ANALYSIS AND INTERPRETATION

4.1 Introduction

This chapter presents the findings of the study. The data relied on was secondary data from a Kenyan insurance company which has been in operation for 15 years. The nature of data was the claims paid by the company for the motor vehicle line of business in 2010. The motor vehicle line of business consists of motor private and motor commercial lines. Data analysis was conducted through the R-statistical programme together with Microsoft Excel. Frequency tables and graphs were used to display the results. Computational results were also used to represent the results.

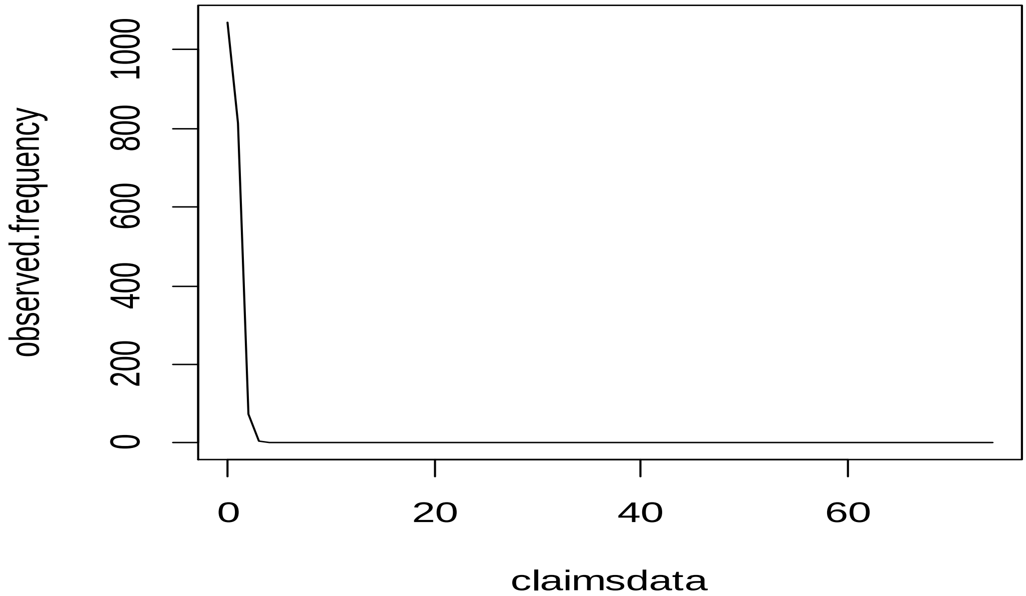
4.2 General Information

4.2.1 Number of claims/ Frequencies

Number of claims	**Observed Frequency**
0	1,071
1	815
2	72
3	3
4	1
5	2
6	1
7	1
8	1
9	1
10+	6
Total	1,974

Table 4.1: Number of claims

Table 4.1 represents the number of claims vis a vis the observed frequency. 1,974 policies were issued in 2010. Out of these, 1,071 made no claims in that year, whereas there were 815 claims which were made once. A graphical representation is shown below:



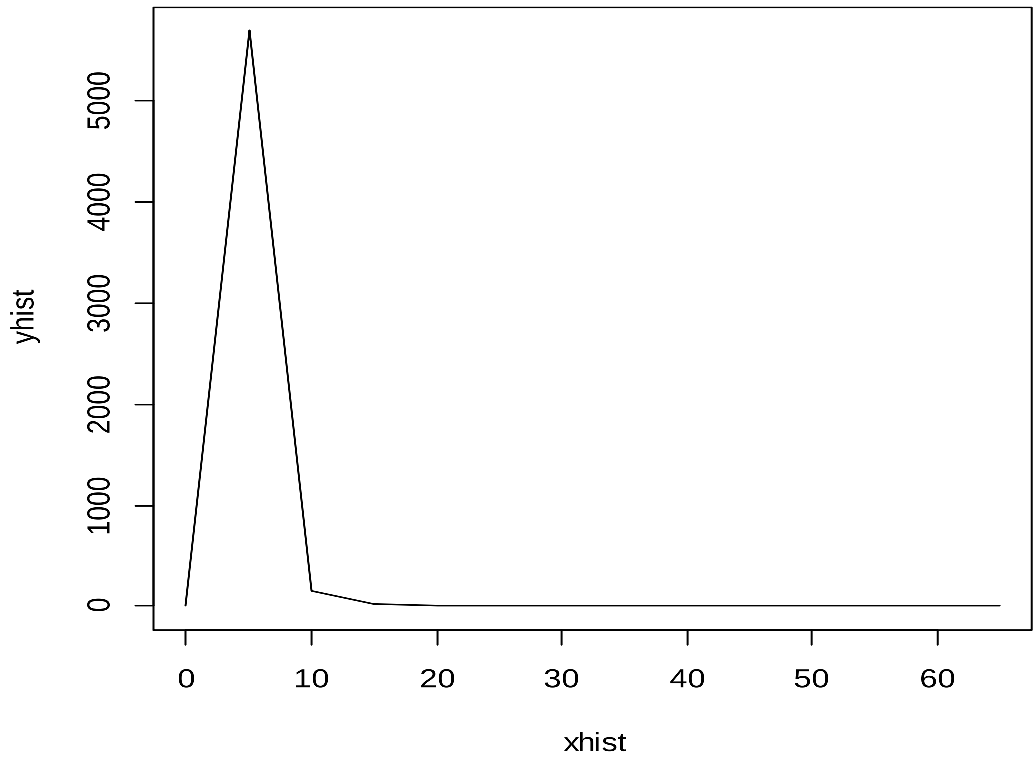
The claims frequencies indicate a trend that reduces drastically as the number of claims increases. The mean of the claims frequencies is 0.6044 and the variance is 4.56.

4.2.2 Claims Severity/ Amounts

Claims Amounts	Claims Frequency	Midpoint ,m	m*f	m-ḿ	$(m-ḿ)^2$	$f*(m-ḿ)^2$
0-500,000	5,686	2.5	14,215	(0.24)	0.06	314.40
500,000-1,000,000	161	7.5	1,207.5	7.50	56.25	9,056.25
1,000,000-1,500,000	25	12.5	312.5	12.50	156.25	3,906.25
1,500,000-2,000,000	15	17.5	262.5	17.50	306.25	4,593.75
2,000,000-2,500,000	1	22.5	22.5	22.50	506.25	506.25
2,500,000-3,000,000	1	27.5	27.5	27.50	756.25	756.25
3,000,000-3,500,000	0	32.5	0	32.50	1,056.25	0
3,500,000-4,000,000	0	37.5	0	37.50	1,406.25	0
4,000,000-4,500,000	0	42.5	0	42.50	1,806.25	0
4,500,000-5,000,000	0	47.5	0	47.50	2,256.25	0
5,000,000-5,500,000	0	52.5	0	52.50	2,756.25	0
5,500,000-6,000,000	0	57.5	0	57.50	3,306.25	0
6,000,000-6,500,000	1	62.5	62.5	62.50	3,906.25	3,906.25
Total	**5,890**		**16,110**			**23,039.40**
		mean	2.74		variance	3.91

Table 4.2: Claims severities/ claims amount

Table 4.2 represents the claims amounts alongside the frequencies of occurrence of such claims. 5,686 claimants made claims of between Kes 0 to Kes 500,000. For ease of computation, the claims amounts have been divided by Kes 100,000. A graphical representation is shown below:

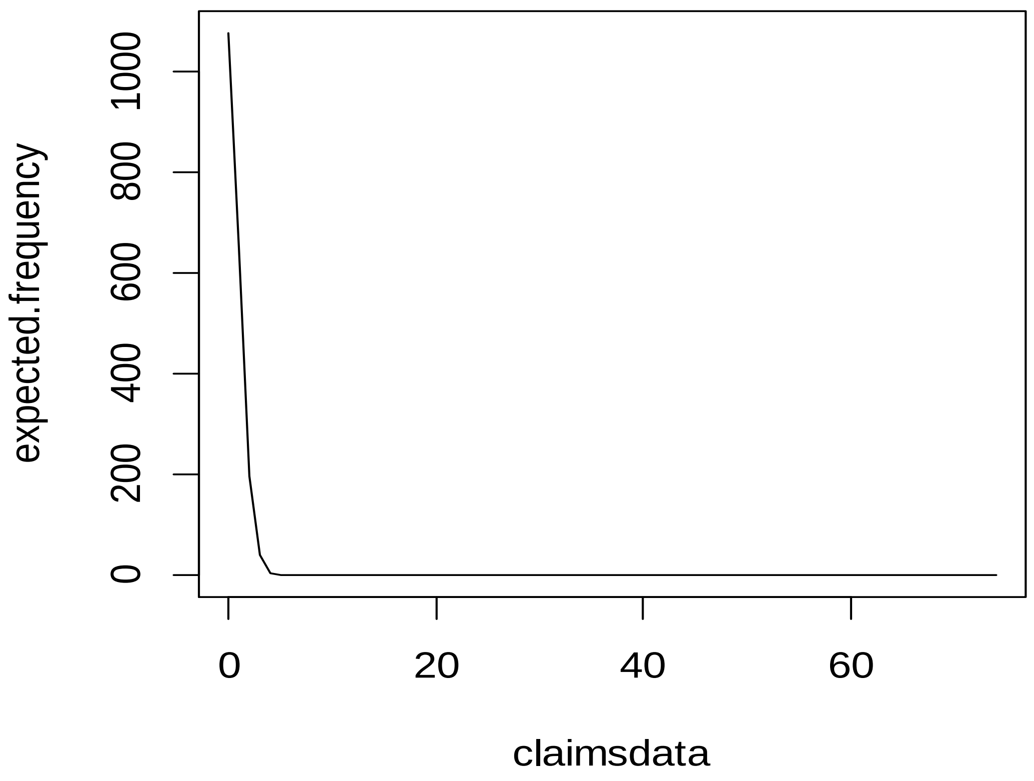


4.3 Fitting of the data

4.3.1 Fitting the number of motor vehicle claims data

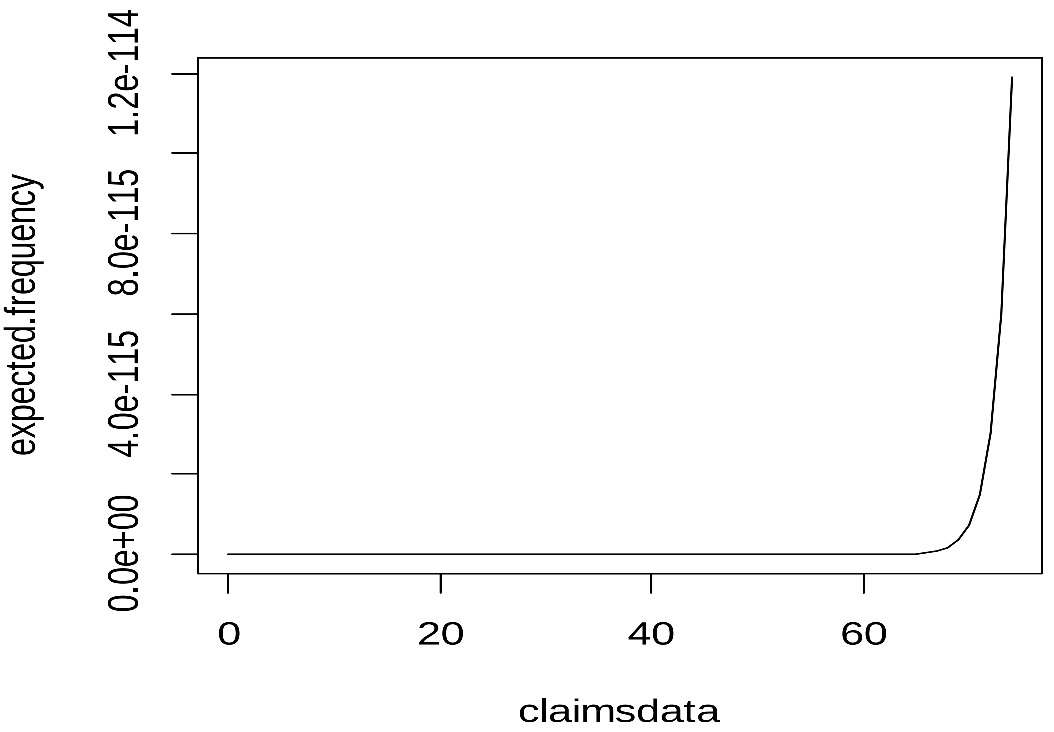
The motor vehicle claims data is fitted to various distributions in order to determine the distribution that would be suitable for modeling such data.

A graphical representation of the expected frequencies fitted using the Poisson distribution is shown below:



Graphically, the fitted data seems to follow the same trend as the observed data. However, a goodness-of-fit test is necessary to statistically evaluate the fitted data.

A graphical representation of the expected frequencies fitted using the negative binomial distribution is shown below:



Graphically, the fitted data does not seem to follow the same trend as the observed data. However, a goodness-of-fit test is necessary to statistically evaluate the fitted data.

4.3.2 Goodness-of-Fit tests for the number of motor vehicle claims data

A χ^2 goodness of fit test was conducted with the following hypothesis:

> H_o: The Poisson distribution provides a good fit for the number of claims
>
> H_1: The Poisson distribution does not provide a good fit for the number of claims.

The results obtained indicated that the test-statistic χ^2 = 4.687791e+120 = 4.69* 10^{120} and the degrees of freedom = 74, the table value = 0. This implies that we reject the null hypothesis and conclude that the Poisson distribution does not provide a good fit for the number of claims.

A second χ^2 goodness of fit test was conducted with the following hypothesis:

H_o: The Negative Binomial distribution provides a good fit for the number of claims

H_1: The Negative Binomial distribution does not provide a good fit for the number of claims.

The results obtained indicated that the test-statistic χ^2 = 1.246082e+163 = $1.246*10^{163}$; the degrees of freedom were 74 and the table value was equal to zero. We thus reject the null hypothesis and conclude that the Negative Binomial Distribution does not provide a good fit for the number of claims.

However, despite the goodness of fit test failing in all respects, we opt for a fit that is better than the other one. The Poisson fit provides a better fit as it is closer to the region where we fail to reject the null hypothesis.

A study conducted by Bastida and Sanchez (2013) reveals that the Poisson-Lindley model provides a sound fit for the number of claims. Infact, at 5% level of significance, it is noticeable that for the given data set, the Negative Binomial Distribution and the Poisson distribution provide a good fit. The number of claims are thus concluded to follow a Poisson distribution.

4.3.3 Fitting the claim amounts per loss event

The motor vehicle claims amounts are fitted to various continuous distributions in order to determine the most suitable distribution to model the data. For ease of computation, the claims amount data will be represented as divided by 100,000. The gamma distribution is fitted to the data and various parameters are computed as follows:

i. The lambda estimate given by the R-code l.est=med.gamma/var.gamma is found to be 0.6991309

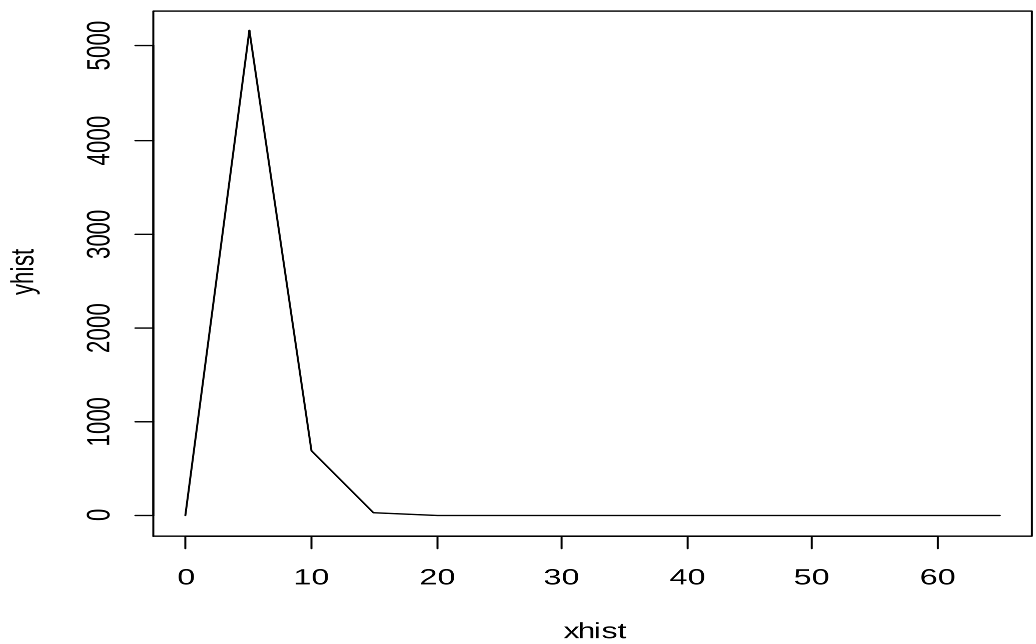
ii. The alfa estimate given by the R-code a.est=((med.gamma)^2/var.gamma) is found to be 1.1912123

The expected claims frequencies obtained are shown in the table below:

Claims Amounts	**Claims Frequency**	**Expected Frequency**
0-500,000	5,686	5,162.7830000
500,000-1,000,000	161	689.8990000
1,000,000-1,500,000	25	35.7407700
1,500,000-2,000,000	15	1.5153000
2,000,000-2,500,000	1	0.0587000
2,500,000-3,000,000	1	0.0021580
3,000,000-3,500,000	0	0.0000767
3,500,000-4,000,000	0	0.0000027
4,000,000-4,500,000	0	0.0000001
4,500,000-5,000,000	0	0.0000000
5,000,000-5,500,000	0	0.0000000
5,500,000-6,000,000	0	0.0000000
6,000,000-6,500,000	1	0
Total	5,890	

Table 4.3: Expected claims severities modeled using the gamma distribution

A graphical representation of the fitted data is shown below:



The data was also fitted into the exponential distribution and the following estimates were used:

i. The lambda estimate given by the R-code l.est=med.gamma/var.gamma is found to be 0.6991309

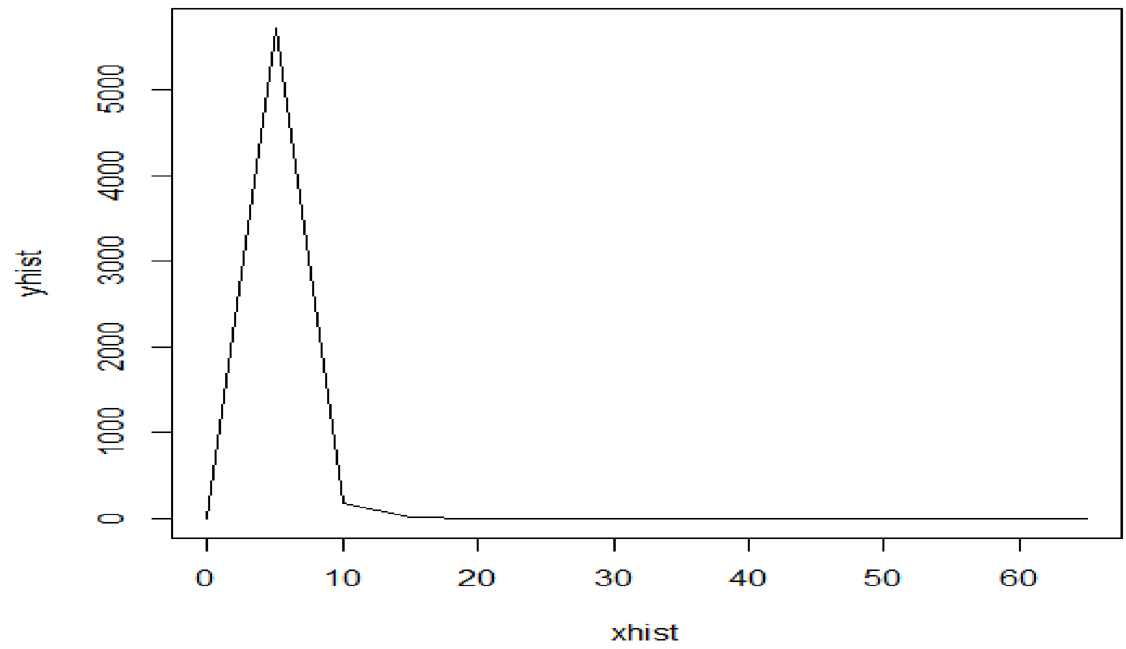
ii. The alfa estimate used is 1, the exponential distribution is a special case of the gamma distribution which assumes that the alfa is equal to 1

The expected frequencies obtained are shown in the table below:

Claims Amounts	**Claims Frequency**	**Expected Frequency**
0-500,000	5,686	5,711.3630000
500,000-1,000,000	161	173.2193000
1,000,000-1,500,000	25	5.2536000
1,500,000-2,000,000	15	0.1593330
2,000,000-2,500,000	1	0.0048320
2,500,000-3,000,000	1	0.0001465
3,000,000-3,500,000	0	0.0000044
3,500,000-4,000,000	0	0.0000001
4,000,000-4,500,000	0	0.0000000
4,500,000-5,000,000	0	0.0000000
5,000,000-5,500,000	0	0.0000000
5,500,000-6,000,000	0	0
6,000,000-6,500,000	1	0
Total	5,890	

Table 4.4: Expected claims severities modeled using the exponential distribution

A graphical representation is shown below:



4.3.4 Goodness-of-Fit tests for the claims amount per loss event

A chi-square goodness of fit test was conducted with the following hypothesis:

H_0 : The data comes from the stated distribution

H_1: The data does not come from the stated distribution

The last four classes is clumped together to enable easier analysis using the chi-square. The result obtained for the gamma distribution are:

Chi-squared test for given probabilities

data: observed.frequency
X-squared = 1836.171, df = 4, p-value < 0.022

The p-value tends to be greater than the alfa at 0.01 level of significance hence the null hypothesis is not rejected.

A second fit for the exponential distribution is done and the following results obtained:

Chi-squared test for given probabilities

data: observed.frequency
X-squared = 3257.414, df = 4, p-value < 0.022

The p-value tends to also be greater than alfa at 0.01 level of significance and hence the null hypothesis is not rejected.

Since both the gamma and exponential distribution give good fits for the claims severity, we are indifferent. For this study, we select the exponential distribution since it has an alfa equal to one and produces a higher X-squared. A higher X-squared gives the impression of randomness of the given data.

4.4 Discretizing the claims severity

The claims severity, X is discretized using the method of rounding. As earlier said, the claims severity exhibits the exponential distribution which is continuous. The discretization is done by applying the general formulae:

$$f_j = F(2j+1) - F(2j-1) \text{ for j} \geq 1\text{; and;}$$

$$f_0 = F(1) \text{ for j =0}$$

An assumption that the motor vehicle claims are only paid if they are in excess of Kes 20,000. In this computation, the Kes 20,000 is represented by 0.2. The exponential cumulative density function (CDF) of an exponential distribution is given by $1-e^{-\lambda x}$. It would be of essence to compute an adjusted lambda,

$$\lambda^* = v\lambda.$$

$$v = \Pr ob(X > 0.2)$$

$$\Pr ob(X > 0.2) = 1 - \Pr ob(X < 0.2)$$

$$\Pr ob(X > 0.2) = 1 - \Pr ob(X < 0.2)$$

$$\text{V= } \Pr ob(X > 0.2) = 1 - \left\{1 - \left(e^{-0.6991*0.2}\right)\right\} = 0.8695$$

$$\lambda^* = 0.8695*0.6991=0.6078775$$

The discretized values are found in the table below and the span used to discretize the data is h=5.

Claims Amounts	**J**	**f_j=Probabilities**
0	0	0.455494644
5	1	0.383067081
10	2	0.113574001
15	3	0.033673094
20	4	0.009983599
25	5	0.002959997
30	6	0.000877597
35	7	0.000260195
40	8	7.71442E-05
45	9	2.28722E-05
50	10	6.78128E-06
55	11	2.01055E-06
60	12	5.96101E-07
65	13	1.76736E-07

Table 4.5: Discretized claims severities probabilities

4.5 Computing the aggregate probabilities

The aggregate probabilities are determined using the recursive formulae given below:

$$p_k = (a + (b / k))\, p_{k-1}$$

Using the case of Poisson distribution $f_S(0) = \left(e^{-\lambda[1-f_x(0)]}\right)$, the initial aggregate probability is computed. This enables us to get the aggregate payments distribution at time 0. To determine the subsequent probabilities we use the recursive formulae below:

$$f_S(x) = \frac{\lambda}{x} \sum_{y=1}^{x \wedge r} y * f_x(y) * f_S(x-y)$$

Claims	K	f_k	fs(x)
0	0	0.455494644	0.846619183
500,000	1	0.383067081	0.099171348
1,000,000	2	0.113574001	0.03284711
1,500,000	3	0.033673094	0.010076929
2,000,000	4	0.009983599	0.003018432
2,500,000	5	0.002959997	0.000897655
3,000,000	6	0.000877597	0.000266383
3,500,000	7	0.000260195	7.9E-05
4,000,000	8	7.71442E-05	2.34242E-05
4,500,000	9	2.28722E-05	6.94512E-06
5,000,000	10	6.78128E-06	2.05915E-06
5,500,000	11	2.01055E-06	6.10509E-07
6,000,000	12	5.96101E-07	1.81007E-07
6,500,000	13	1.76736E-07	5.36662E-08

Table 4.6: Table showing the computed aggregate probabilities

4.6 Summary of Findings

The expected payment per loss greater than Kes 20,000 as given by the exponential distribution is given as Kes 386,989.04

The expected number of losses per policyholder is 0.6044. The expected number of losses for all policyholders in total is given by 0.6044*903 = 545.77

The expected aggregate losses in payments greater than 20,000 per individual is given by 386,989.04*0.6044 = 233,896.12

The expected aggregate loss found for the insurance company in relation to the motor vehicle claims is given as Kes 211,208,192.12.

4.7 Determining premiums using the expected value principle

Using the expected value principle: $(1+\theta)E(S)$ and assuming a premium loading of 0.02, then the premiums that should be paid for the following year should be

$$
\begin{aligned}
&= 1.02 * 211208192.12 \\
&= 215,432,355.96 \\
&= 215,432,355.96 / 1974 \\
&= 109,134.93
\end{aligned}
$$

This figure translates to an average of Kes 109,134.93 per individual who is insured. This figure is relatively high compared to that which would have been determined through the rate pricing of premiums. Under rate pricing, assuming a vehicle which is valued at Kes 1,000,000 would require an insurance premium of Kes 40,000. Our pricing model gives a figure which is 63% higher than the figure which would have been achievable through rate pricing.

4.8 Determining premiums using the standard deviation principle

Using the standard deviation principle $E(S)+\theta\sqrt{Var(S)}$ and assuming the same premium loading of 0.02 as above, then the premiums that should be paid for the following year should be:

$$
\begin{aligned}
&= 233{,}896.12 + 0.02 * \sqrt{35{,}807{,}861{,}111.53} \\
&= 237{,}680.71
\end{aligned}
$$

In comparison to the rate pricing, the premium obtained is much higher.

4.9 Determining the estimated credibility premium for the number of claims

Using the data that was obtained in Table 4.1 above, we denote that we have a total of 1,974 policyholders which is our r variable. We denote a 1 year past experience on each of the policy holders ie n_i=1.The exposures, m_{ij} are also equal to 1.

For policyholder i, where (i= 1,….., 1974), we make an assumption that $X_{i1} \mid \theta_i = \theta_i$ is Poisson distributed with a mean of θ_i such that $\mu(\theta_i) = \nu(\theta_i) = \theta_i$.

$$
\mathrm{X}(bar) = \frac{1}{1974} * \left[\sum_{i=1}^{1974} X_{i1} \right],
$$

$$
\mathrm{X}(bar) = \frac{1}{1974}[1193] = 0.604357
$$

$$
Var(X_{i1}) = \frac{1}{n-1} * \sum_{i=1}^{1974} \left(X_{i1} - X(bar) \right)^2
$$

$$
Var(X_{i1}) = \frac{1}{1973} * 8998 = 4.56
$$

claims no	frequency	Claims*frequency	f*(claims no - mean claims)^2
0	1071	0	391.18
1	815	815	127.57
2	72	144	140.24
3	3	9	17.22
4	1	4	11.53
5	2	10	38.64
6	1	6	29.11
7	1	7	40.90
8	1	8	54.70
9	1	9	70.49
10	1	10	88.28
11	0	0	-
12	1	12	129.86
13	0	0	-
14	0	0	-
15	0	0	-
16	0	0	-
17	0	0	-
18	1	18	302.61
19	0	0	-
20	0	0	-
21	0	0	-
22	0	0	-
23	0	0	-
24	0	0	-
25	0	0	-
26	0	0	-
27	0	0	-
28	0	0	-
29	0	0	-
30	0	0	-
31	0	0	-
32	1	32	985.69
33	0	0	-
34	0	0	-
35	1	35	1,183.06
36	0	0	-
72	0	0	-
73	0	0	-

74	1	74	5,386.92
		1193	**8,998.00**

Table 4.7: Table showing the credibility premium computation

The values between 37 and 71 have been omitted due to the values carrying zero observations. They have however been used for all the computations.

$$Var(X_{i1}) = Var\left[E(X_{i1} \mid \theta_{i1})\right] + E\left[Var(X_{i1} \mid \theta_i\right]$$

$$= Var\left[\mu(\theta_i)\right] + E\left[Var(\theta_i)\right]$$

But $a = Var\left[\mu(\theta_i)\right]$ and $v = E\left[Var(\theta_i)\right]$ thus we get that

$$Var(X_{i1}) = a + v \ \ = a + \mu$$

An unbiased estimator for a and v is the sample variance. The following observations are made:

$$\hat{a} = 4.56 - 0.604357 = 3.955643$$
$$ķ = \frac{0.604357}{3.955643} = 0.152783504$$
$$Z = \frac{1}{1 + 0.152783504} = 0.86746557$$

These values are fitted into the partial credibility premium formulae given as $P_C = ZX(bar) + (1 - Z)M$

The estimated credibility premium for the number of claims for each policyholder is given by (0.87)* X_{i1}+(0.13)*(0.604357) where X_{i1}=0,1,2,…..,74 depending on the policyholder.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

The study reveals a great difference in pricing between the risk modeling measures and the rate pricing measures. The expected value principle and standard deviation principles lead to a premium value that is over 63% higher than the rate pricing method. Despite the premium amount being higher than the rate pricing amount, the level of risk resulting from underwriting is better handled.

5.2 Conclusion

From the findings above, we conclude that risk modelling as a risk management strategy yields higher premiums for the insurer. These higher premiums enable the underwriter to comfortably cover their claims when they arise.

Despite the cut-throat competition in the insurance industry, the risk based pricing would appear more practical when claims are being paid.

5.3 Limitations of the Study

There lacked enough local as well as international studies on underwriting risk. The study relied on a comparison between the risk management strategies in Kenya as well as those abroad.

Secondary data was used to conduct the research. This type of data could have a lot of errors due to smoothing techniques being applied on it. The data may not represent the true feeling on the ground.

5.4 Suggestions for further research

There is need for further research especially in the line of reserving for aggregate claims. Creation of reserves is an important component of underwriting risk. Inadequate reserves set aside expose the insurance company to losses in future years. This is because the amount set aside to cover future risk will be much lower than what is required.

Another area of research is the use of a mixture model to fit the number of claims. Mixtures of the Poisson distribution have been seen to give more accurate fits to the discrete random variables.

It would also be prudent to include the effects of inflation to the computation of premiums. Inflation has been on the upward trend in the last five years. Premiums should reflect inflation as this gives a complete economic outlook.

A similar study should be conducted using a larger sample and a change in the model used for example simulation methods.

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APPENDIX ONE

6.0 R – codes used

6.1 Codes used to represent the claims data:

```
claimsdata=seq(0,74,1)

claimsdata

observed.frequency=c(1071,815,72,3,1,2,1,1,1,1,1,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0
,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1)

observed.frequency
```

6.2 Codes used to fit the claims data using Poisson probabilities

```
sum(observed.frequency) =1974

product=claimsdata*observed.frequency

product

sum(product) =1193

mean=sum(product)/sum(observed.frequency)

mean = 0.6043566

expected.frequency=c(dpois(claimsdata,lambda=mean)*1974)

expected.frequency

plot(claimsdata,expected.frequency,'line')

plot(claimsdata,observed.frequency,'line')
```

6.3 Codes used to fit the claims data using the negative binomial probabilities

```
claimsdata=seq(0,74,1)

claimsdata

observed.frequency=c(1071,815,72,3,1,2,1,1,1,1,1,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0
,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1)

observed.frequency

sum(observed.frequency)

product=claimsdata*observed.frequency

product

sum(product)
```

```
mean=sum(product)/sum(observed.frequency)

mean

expected.frequency=c(dnbinom(claimsdata,p=0.008058088,size=75)) #since the number of observations is 75 and the mean is 0.6043566,then p=0.008058088

expected.frequency

plot(claimsdata,expected.frequency,'line')
```

6.4 Testing the goodness of fit

```
x2<-sum(((observed.frequency-expected.frequency)^2)/expected.frequency) #test statistic

x2

gdl<-74

1-pchisq(x2,gdl)
```

6.5 Codes used to represent the claims data

```
claims.amount=c(0,5,10,15,20,25,30,35,40,45,50,55,60,65)

claims.amount

length(claims.amount)

observed.frequency=c(5686,161,25,15,1,1,0,0,0,0,0,0,1)

observed.frequency

h=hist(claims.amount,breaks=13)

xhist=c(min(h$breaks),h$breaks)

length(xhist)

yhist=c(0,observed.frequency)

length(yhist)

plot(xhist,yhist,'line')
```

6.6 Fitting the gamma distribution to obtain the expected frequencies and representing it graphically

```
claims.amount=c(0,5,10,15,20,25,30,35,40,45,50,55,60,65)

claims.amount

claims.amount.cut=cut(claims.amount,breaks=c(0,5,10,15,20,25,30,35,40,45,50,55,60,65))

claims.amount.cut

observed.frequency=c(0,5686,161,25,15,1,1,0,0,0,0,0,0,1)
```

```
observed.frequency

med.gamma=mean(observed.frequency)

med.gamma

var.gamma=var(observed.frequency)

var.gamma

l.est=med.gamma/var.gamma

l.est

a.est=((med.gamma)^2/var.gamma)

a.est

(pgamma(5,shape=a.est,rate=l.est)-pgamma(0,shape=a.est,rate=l.est))*5890

(pgamma(10,shape=a.est,rate=l.est)-pgamma(5,shape=a.est,rate=l.est))*5890

(pgamma(15,shape=a.est,rate=l.est)-pgamma(10,shape=a.est,rate=l.est))*5890

(pgamma(20,shape=a.est,rate=l.est)-pgamma(15,shape=a.est,rate=l.est))*5890

(pgamma(25,shape=a.est,rate=l.est)-pgamma(20,shape=a.est,rate=l.est))*5890

(pgamma(30,shape=a.est,rate=l.est)-pgamma(25,shape=a.est,rate=l.est))*5890

(pgamma(35,shape=a.est,rate=l.est)-pgamma(30,shape=a.est,rate=l.est))*5890

(pgamma(40,shape=a.est,rate=l.est)-pgamma(35,shape=a.est,rate=l.est))*5890

(pgamma(45,shape=a.est,rate=l.est)-pgamma(40,shape=a.est,rate=l.est))*5890

(pgamma(50,shape=a.est,rate=l.est)-pgamma(45,shape=a.est,rate=l.est))*5890

(pgamma(55,shape=a.est,rate=l.est)-pgamma(50,shape=a.est,rate=l.est))*5890

(pgamma(60,shape=a.est,rate=l.est)-pgamma(55,shape=a.est,rate=l.est))*5890

(pgamma(65,shape=a.est,rate=l.est)-pgamma(60,shape=a.est,rate=l.est))*5890

expected.frequency=c(0,5162.78,689.9,35.74,1.515,0.059,0.0021,0.0000767,0.000002664,0.
0000000909,0.00000000306,0.000000000102,0.0000000000327,0)

expected.frequency

h=hist(claims.amount,breaks=13)

xhist=c(min(h$breaks),h$breaks)

length(xhist)

yhist=c(0,expected.frequency)
```

```
length(yhist)

plot(xhist,yhist,'line')
```

6.7 Fitting the exponential distribution to obtain the expected frequencies and representing it graphically

```
claims.amount=c(0,5,10,15,20,25,30,35,40,45,50,55,60,65)

claims.amount

claims.amount.cut=cut(claims.amount,breaks=c(0,5,10,15,20,25,30,35,40,45,50,55,60,65))

claims.amount.cut

observed.frequency=c(0,5686,161,25,15,1,1,0,0,0,0,0,0,1)

observed.frequency

med.gamma=2.735

var.gamma=3.912

var.gamma

l.est=med.gamma/var.gamma

l.est

a.est=1

a.est

(pgamma(5,shape=a.est,rate=l.est)-pgamma(0,shape=a.est,rate=l.est))*5890

(pgamma(10,shape=a.est,rate=l.est)-pgamma(5,shape=a.est,rate=l.est))*5890

(pgamma(15,shape=a.est,rate=l.est)-pgamma(10,shape=a.est,rate=l.est))*5890

(pgamma(20,shape=a.est,rate=l.est)-pgamma(15,shape=a.est,rate=l.est))*5890

(pgamma(25,shape=a.est,rate=l.est)-pgamma(20,shape=a.est,rate=l.est))*5890

(pgamma(30,shape=a.est,rate=l.est)-pgamma(25,shape=a.est,rate=l.est))*5890

(pgamma(35,shape=a.est,rate=l.est)-pgamma(30,shape=a.est,rate=l.est))*5890

(pgamma(40,shape=a.est,rate=l.est)-pgamma(35,shape=a.est,rate=l.est))*5890

(pgamma(45,shape=a.est,rate=l.est)-pgamma(40,shape=a.est,rate=l.est))*5890

(pgamma(50,shape=a.est,rate=l.est)-pgamma(45,shape=a.est,rate=l.est))*5890

(pgamma(55,shape=a.est,rate=l.est)-pgamma(50,shape=a.est,rate=l.est))*5890

(pgamma(60,shape=a.est,rate=l.est)-pgamma(55,shape=a.est,rate=l.est))*5890
```

```
(pgamma(65,shape=a.est,rate=l.est)-pgamma(60,shape=a.est,rate=l.est))*5890

expected.frequency1=c(0,5711.363,173.2193,5.2536,0.15933,0.004832,0.0001465,0.000004445,0.0000001348,0.000000004088,0.0000000001242,0.000000000003924,0,0)

expected.frequency1

h=hist(claims.amount,breaks=13)

xhist=c(min(h$breaks),h$breaks)

length(xhist)

yhist=c(0,expected.frequency1)

length(yhist)

plot(xhist,yhist,'line')
```